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Characterizing the Interfiber Bonding of Currency Paper Pulps

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Center for Materials Science
Polymer Science and Standards Division
Washington, DC 20234

Final Report Covering the Period

October 1, 1980 through September 30, 1981

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, ***Secretary***
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, ***Director***

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Summary and Conclusions

The purpose of this project has been to study the nature of interfiber bonding in paper and to develop parameters that characterize the interfiber bond strength. Most of the data for these studies were obtained by means of tensile tests on specimens cut from low-density open-web handsheets made from Kraft woodpulp or from currency paper pulp. These tests were expected to provide information of use in attempts to develop a durable currency paper made from woodpulps.

In a typical test a specimen 1 cm wide by 2 cm long was extended in a sensitive tensile tester. On the resulting chart record the force rises and dips through a series of peaks, and eventually diminishes to zero. Each force drop, or jag in the curve, is assumed to indicate the breaking of a bond between fibers. In previous work on woodpulp specimens three different parameters had been derived from force-elongation data. For one of these parameters the percent elongation of the specimen at each bond break was plotted versus the number of bond breaks. The initial slope giving the average elongation between breaks was used as a parameter to characterize the amount of bonding per unit area of specimen.

For a second parameter the force drops incurred by the specimen in a series of breaks were averaged to obtain a quantity characterizing the average interfiber bond strength. In a third parameter the energy loss incurred by the specimen in each of a series of breaks was calculated, and these energy losses were averaged to obtain a quantity characterizing the average interfiber bond strength. During the past year the rationale for these latter two parameters was reexamined. It was found that because of the dependence of the force drop and energy loss upon the size of the hole opened up by the bond break it was necessary to impose restrictions on the way the averaging process was carried out. It was also necessary to obtain by an independent measurement a parameter characterizing the average mesh size (distance between bonds on a fiber segment) of the specimen network tested.

The first parameter had been evaluated for Northern and Southern Kraft woodpulps and currency pulps, and results were reported previously. The force drop and energy loss parameters were not evaluated, as most of the emphasis during this past year was placed on characterizing the force-elongation curve up to bond break, and in studying how the characteristic constants of this curve changed as the specimen deteriorated during test.

If during a tensile test the extension is reversed until the force drops to zero and the specimen is then reextended, a curve up to the first bond break is obtained for the force-elongation behavior of the specimen at that particular stage of the test. In order to study how the specimen deteriorates during a test a series of these curves is obtained as a part of the testing procedure. According to previous work on woodpulp specimens these curves are well fitted by the equation

$$F = C_2[e^{(x-x_o)/x_c} - 1]$$

Here F is the force and x is an elongation variable. x_o is the value of this variable when the force is zero. x_c is a characteristic length believed to be approximately proportional to the average size of the meshes in the specimen. C_2 is a quantity analogous to a "spring constant"; that is, its value is approximately proportional to the width of the specimen, but it is also sensitive to the structure and force distribution within the specimen.

In order to understand the force-elongation behavior, it is postulated that during extension the force sustained by a specimen is channeled through the fiber segments in a series of interlocking parallel pathways. In addition there are many other fiber segments available that do not sustain force because they have a curled configuration or are not sufficiently oriented in the direction of stretch. However, as the elongation of the specimen is increased these fiber segments successively join with the other segments in the network and add to the force resisting elongation.

The quantity x_c governs the rate at which additional fiber segments sustain part of the force. If x_c is small the exponential term in the force-elongation equation above increases rapidly as x increases. This would be the behavior expected of a network with a small average mesh size, thus the association of x_c with mesh size. The quantity C_2 depends upon the primary structure of the network; that is, upon the number of pathways able to sustain force at the start of the elongation process. This primary structure is maintained intact throughout an extension until the first bond break occurs. The primary structure then changes slightly and the effect of this change is evinced by a change in the value of C_2. Bond breaks also cause changes in the value of x_c, but these changes are not so pronounced as the changes in C_2.

As the elongation progresses the unstrained length of the specimen increases. This increase in length is made manifest by an increase in the value of x_o; thus, for a given reextension curve the unstrained length ℓ is equal to $\ell_o + x_o - x_s$ where ℓ_o is the original unstrained length of the specimen, and x_s is the value of x_o for the initial force-elongation curve. The values of ℓ for a series of reextension curves obtained during a test can be used to characterize the progressive deterioration of the specimen.

As part of the experimental work during this past year, representative values of the quantities C_2 and x_c were obtained for a number of Northern and Southern Kraft woodpulp handsheets. For each specimen tested successive values of characteristic strain x_c/ℓ were plotted versus the fractional increase in specimen unstrained length $\ell/\ell_o - 1$. A reference value $(x_c/\ell)_o$ was then obtained by a linear extrapolation to $\ell/\ell_o - 1 = 0$. It was found that C_2 values decreased very rapidly as the specimen deteriorated during a test, so in this case values of $\log_e C_2$ were extrapolated to find a reference value of $(\ln C_2)_o$.

A short series of tests was made to see if the values of C_2 and x_c depended upon the shape of the specimen. Specimens of dimensions 1x2, 1x5 and 2x5 cm-width x length were used in these tests. It was found, as expected, that values of $(x_c/\ell)_o$ and $(\ln C_2)_o$ for the larger area specimens had less scatter. Evidently textural inhomogeneities in the handsheets had dimensions of the order of 1 cm. In all of the other tests 1x2-cm specimens were used, although in the light of these results a 1x5-cm specimen might have been a better choice. A 2x5-cm specimen provides even less scatter in values of C_2 and x_c, but at considerable sacrifice in sensitivity in the measure of force drop or energy loss per break.

According to the previous discussion it is expected that C_2 should increase as the density of the handsheet is increased. In order to test this, values of $(\ln C_2)_o$ were plotted versus area density for a series of Northern woodpulp handsheets of density values varying between 1.5 and 3.5 g/m^2. It was found that $(\ln C_2)_o$ increased in an approximately linear fashion with density. Values of $(x_c/\ell)_o$ were plotted versus area density for the same series of handsheets. $(x_c/\ell)_o$ was found to decrease in an approximately linear fashion with density, as was expected. Similar results for C_2 and x_c were obtained in tests for a series of handsheets of Southern Kraft woodpulp.

The parameters $(\ln C_2)_o$ and $(x_c/\ell)_o$ were also determined for a series of handsheets of the same density prepared from Northern Kraft woodpulp beaten various amounts in a laboratory beater. It was expected that increased amounts of beating might increase the number of bonds per unit area, resulting in an increase in the value of $(\ln C_2)_o$ and a decrease in the value of $(x_c/\ell)_o$. These expectations were confirmed. Similar confirmation resulted from tests on Southern Kraft woodpulp handsheets.

Although these test results were in accordance with expectations, the scatter in specimen-to specimen values was very large. This scatter may be attributed to the textural inhomogeneity of the handsheets. Some variation results from differences in the average density of the test specimen, but serious imprecision may result from density fluctuations in the test specimen. During a test the distribution of forces in the specimen may change so that at different times a major portion of the load is shifted from one portion of the specimen to another, thus the values of C_2 and x_c measured from the reextension curves will fluctuate in accordance with variations in local density of the test specimen. Under these conditions the extrapolated values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ are not very precise.

Because of the difficulty in obtaining good reference values of C_2 and x_c for a handsheet or for a test specimen, it may be prudent to change the method for calculating bonding parameters. Instead of evaluating a parameter for each test specimen by using average values of C_2 and x_c in the calculation, it may be more desirable to evaluate subparameters for different stages of a test, using the changing values of C_2 and x_c in the calculation. These subparameters could be averaged in a suitable fashion to obtain a representative value for the specimen.

According to the results just given the force-elongation behavior of a woodpulp specimen can be satisfactorily represented by a single exponential equation having as parameters the quantities C_2, x_c and x_o. For currency pulp specimens, however, this equation is found to be inadequate. When a currency pulp specimen is extended it behaves as though there were two mechanisms operating in parallel; a strong spring-like force that dominates the initial stages of the extension, and a force mechanism similar to that operating in woodpulp specimens that dominates the later stages of the extension. The strong spring-like force might simulate the effect required to orient a fibrous network partially immobilized by numerous adhesions between small fibrils and by the cementing action of fines. The initial strong spring-like force might also result from

an initial maldistribution of forces in which the force resisting extension is sustained by only a few highly extended fiber pathways. Thus the force-elongation behavior can be expressed by an equation of the type

$$F = k(x - x_o) + C_2[e^{(x-x_o)/x_c} - 1]$$

where k is the spring constant of the spring-like force.

The force-elongation behavior of a large number of currency pulp specimens has been analyzed using the equation just given. In most instances significant values of k were required to obtain a good fit to the data. This suggests that much of the binding in currency pulps is achieved by other means than simple bonds at fiber junctions. Thus in this case the concept of an interfiber bond strength characterized by force drop or energy loss parameters may not be appropriate.

Although for currency pulp paper it might be necessary to discard the concept of average interfiber bond strength, a suitable characterization of the overall ability to hold together might yet be possible. Force level and elongation parameters have been tentatively proposed for this purpose. There parameters were developed for wood pulp papers, but their generalization for use with currency pulp papers seems possible. To find a force level parameter for a test specimen, the average force F_{bi} at which bonds break is measured in the vicinity of each reextension curve. These values are then reduced to the values F_{sbi} that would be obtained if the force-elongation behavior of the specimen were governed by coefficients having the values C_{2s} and x_{cs}. The force level parameter is the average value of the F_{sbi}. To find an elongation parameter, the specimen strain ϵ_{bi} at which bonds break is calculated for each F_{bi}. The elongation parameter for the specimen is the average of the ϵ_{bi}. The force level and elongation parameters have not been adequately evaluated from test data, so their applicability and usefulness have not been established.

Review and Discussion of Some Previous Ideas

In this research tensile tests have been emphasized as a means of characterizing bonding between paper pulp fibers. The test specimens are cut from low-density open-web handsheets made from the pulp to be evaluated. The force-elongation curve for such a web specimen has a jagged appearance similar to that shown in figure 1. The force is seen to rise and dip through a series of peaks, and eventually diminish to zero. Each force drop, or jag in the curve, is assumed to indicate the breaking of a bond between fibers. Thus force-elongation curves should provide information that could be used to characterize bond strength.

The handsheets tested, however, have a very complex structure. The pulp fibers have a distribution of lengths and diameters, and may have straight or curled configurations. The interfiber bonds at the fiber crossovers have a broad distribution in strength. The fibrous network of the specimen consists of meshes of a large variety of shapes and sizes. The density of the handsheet fluctuates from spot to spot along its surface. In this circumstance any generalization about the texture of the specimen or its tensile behavior is true only to a rough approximation. The validity of any parameter for characterizing average bond strength therefore must be established empirically.

1. Amount of Bonding per Unit Area

If, in a tensile test, the average elongation between successive jags is small, the web has a relatively large number of fiber crossovers per unit area, and most of these crossovers are bonded. On the other hand, if the average percent elongation per break is large, the web has only a few interfiber bonds per unit area. This means that there is a low density of crossovers, or that many of the crossovers are not bonded or both. In order to characterize the number of bonded crossovers per unit area, the percent elongation of the web at each break is plotted versus the number of bond breaks, and the initial slope of this curve is used as the appropriate parameter. This parameter has been evaluated for a number of woodpulp and currency pulp specimens [1 - 4][1]. The method has been discussed in a previous report [5].

1. Figures in brackets indicate references at the end of this report.

2. The Force Drop Parameter

Each time a bond breaks as the network is being extended the force drops abruptly, indicating that some of the fibers are no longer carrying load. It would seem intuitively that the magnitude of this force drop is related to the average level of the interfiber bond strength in the network; thus the average of a series of force drops should provide a parameter for characterizing bond strength. However, in order to develop this parameter the rationale involved must be examined in more detail.

Assume that the network is composed of randomly distributed fibers joined together at the crossovers by bonds, each of which has the same strength. Such a network would consist of a variety of meshes of various shapes and sizes. The fiber segments between bonds would have a distribution of lengths in straight or curled configurations. When the network is extended the fiber segments between bonds would tend to orient in the direction of stretch, and some of these segments would straighten out and sustain force. The force sustained by the entire network would be channeled along a number of parallel interlocking pathways. The forces in the fiber segments, however, would vary greatly from segment to segment because of the different orientations and because of inhomogeneities in the network structure. Many fiber segments would not sustain any force at all because their curled configurations could not be straightened out until bonds are broken in adjacent load bearing segments. This situation provides a considerable redundancy in fibers available to bear load, and suggests that the force-elongation behavior of the network might not be noticibly affected after a series of random bond breaks.

When a bond breaks the force borne by the two segments involved is redistributed through adjacent segments. The size of the associated force drop would seem to depend upon the number of load bearing segments in the vicinity of the break that are appreciably affected. This in turn would depend upon the size of the hole that is formed, or on the size of the meshes in the vicinity of the broken bond. Some model studies [5,6] have suggested that the size of the force drop is directly proportional to the size of the meshes involved in the break.

Assume then that the bond breaks occur at random in the network, and that the size of each associated force drop D is proportional to a length ℓ_m characteristic of the mesh that is open ed up; i.e., D/ℓ_m is a constant independent of mesh size. Thus the distribution of a series of force drops would

be the same as the distribution of mesh sizes in the network, and the average of a series of force drops would be proportional to the average mesh length $\overline{\ell}_m$. Consider a network (1) in which all of the bonds are of strength S_1. Consider also a network (2) composed of fibers arranged exactly the same way as in (1), but whose bonds have a greater strength S_2. Then, if for bond breaks associated with the same mesh size in each network approximately the same fraction of the force is lost, the force drop D_2 will be greater than D_1. It follows that the average of a series of force drops $\overline{D}_2$ in network (2) will be greater than a similar average $\overline{D}_1$ for network (1). It is in this sense that a force drop average $\overline{D}$ can be said to characterize bond strength in a network. If the force drop corresponding to a given mesh size is a monotonic increasing function of bond strength, $\overline{D}_2$ will still be greater than $\overline{D}_1$. This latter assumption is more probable as it is in agreement with some model studies [5,6].

In the discussion so far it has been assumed that the average force drop $\overline{D}$ for a specimen is obtained by averaging all of the force drops in a suitable portion of the force-elongation curve. However, in practise very small force drops cannot be counted because the full-scale setting of the recording instrument may not provide enough sensitivity to detect them, and very large force drops should not be used because, although they occur infrequently, they have too great an effect on the average. Force drops should not be counted until the extending specimen has achieved a reasonably uniform distribution of forces throughout its area, a situation which usually occurs after the first peak force has been attained; nor should force drops be counted after damage accumulation in the specimen has become obvious. Thus the force drops D to be averaged are those occurring in a certain limited sequence, and which exceed a lower value δ and are less than an upper value Δ.

In order to characterize bonding in a given handsheet it is necessary to average the values of $\overline{D}$ obtained for each of the test specimens cut from the handsheet. It will be assumed that bond strength is the same over the entire area of the handsheet, but it cannot be assumed that the average mesh size is uniform over this area. Experimental evidence indicates that a characteristic length x_c, believed proportional to the average mesh size, varies significantly from specimen to specimen in a given handsheet. Because of this variation in average mesh size it is necessary to adjust the values of δ and Δ from specimen to specimen to provide appropriate compensation. The method of measuring the characteristic

length x_c used in making this adjustment is explained in a later section of this report.

Suppose that the average mesh size in a given specimen is characterized by an experimentally determined length x_c, and that an average force drop $\bar{D}$ is found by averaging those force drop values D lying between a lower limit δ and an upper limit Δ. Consider then another specimen (A) from the same handsheet, but having an average mesh size characterized by an experimentally determined length x_{cA}. This suggests that in the frequency distributions of mesh lengths for the two specimens, a mesh length ℓ_{mA} in specimen A corresponds to a mesh length ℓ_m in the other specimen, but differs from it in length by a factor x_{cA}/x_c; i.e., $\ell_{mA} = x_{cA}\ell_m/x_c$. As the bond strengths in the two specimens are the same, the force drops must vary directly as the characteristic lengths; or, $D_A = \ell_{mA}D/\ell_m = x_{cA}D/x_c$. From this one can infer that $\bar{D}_A = x_{cA}\bar{D}/x_c$ if the force drops D_A to be averaged lie between the limiting values $\delta_A = x_{cA}\delta/x_c$ and $\Delta_A = x_{cA}\Delta/x_c$.

In order to obtain a force drop parameter characterizing bond strength in a given handsheet, it is necessary to choose parameters x_c, δ and Δ characterizing a comparison state. $\bar{D}_A$ should then be determined for specimen A by averaging force drops lying between $\delta_A = x_{cA}\delta/x_c$ and $\Delta_A = x_{cA}\Delta/x_c$. Similar values $\bar{D}_B$, $\bar{D}_C$, etc. should be determined for other specimens. The quantities $\bar{D}_A/x_{cA}$, $\bar{D}_B/x_{cB}$, $\bar{D}_C/x_{cC}$ etc. are comparable determinations and can be averaged. The force drop parameter $\bar{\bar{D}}$ characterizing bond strength in the handsheet is given by

$$\bar{\bar{D}} = \frac{x_c}{n} \sum_{i=1}^{n} \frac{\bar{D}_i}{x_{ci}}$$

It should be kept in mind, however, that this parameter is not unique. Many other values of $\bar{\bar{D}}$ can be obtained depending upon the standard parameters x_c, δ and Δ chosen to define it.

In the above discussion it was assumed that each bond in the fiber network had the same strength, but in reality there is a broad distribution of bond strengths. The effect of this broad distribution is mitigated to some extent because in a tensile test the weakest bonds are usually the ones broken. This limits the variation in bond strength affecting the measurement. In developing the rationale for the force drop parameter $\bar{\bar{D}}$ it was necessary to use other questionable assumptions. The validity of this parameter for characterizing bonding can only be established empirically. Force drop parameters $\bar{\bar{D}}$ have not as yet been evaluated for various materials, as current work has been concerned

with methods for evaluating x_c.

3. The Energy Loss Parameter

Each time a bond breaks some of the strain energy stored in the network is dissipated. Thus it would seem that the average energy dissipated per bond break might be used as a parameter to characterize average interfiber bond strength. There are two methods that might be used to obtain this parameter. In the first method the work to extend the specimen from break to break is summed up and plotted against the number of breaks. One then searches for a region of the force-elongation curve in which a series of force drops of approximately the same magnitude occurs, the force level is approximately constant and the specimen is not noticibly deteriorated by the series of bond breaks. Under these conditions it is supposed that there is a one-to-one relationship between force level and the amount of energy stored in the specimen network. Thus if the energy stored remains constant, the work done in extending the specimen over the series of breaks is equal to the energy dissipated by the breaks. The curve of work-to-break versus number-of-breaks should have a slope in this region equal to the energy dissipated per break, which is the desired parameter. A detailed discussion is given elsewhere [1]. Parameters for various woodpulp samples have been evaluated and given in previous reports [2.3]. Results obtained by this method, however, are influenced by the judgement of the observer, and thus may be open to question.

The energy loss for an individual bond break can be calculated from the slope of the force-elongation curve just before a break occurs, and the values of the force just before and just after break. An energy loss parameter can then be obtained by averaging the energy losses for a series of individual bond breaks. Calculation of energy losses for individual bond breaks has been discussed elsewhere [5]. As in the case of force drops, the energy loss for a bond break depends in an approximately linear fashion upon the size of the mesh that is opened up, and nonlinearly upon the strength of the bond that breaks [5,6]. Therefore this energy loss parameter must be obtained by averaging those energy losses lying within certain limits, in a manner similar to that just described for force drops. Energy loss parameters in which those individual energy losses lying within certain limits are averaged, have not as yet been determined for various materials.

4. Other Bonding Parameters

A serious shortcoming of the force drop and energy loss parameters is their dependence on the average mesh size of the specimen networks that are being tested. However model studies [5,6] have suggested that for an individual bond break both the force drop D and the associated energy loss U depend linearly upon the size of the mesh that is opened. D and U also depend upon the bond strength, but the relationship is supposed to be nonlinear and different for the two quantities. Thus the relation U/D should be independent of mesh size but have a nonlinear dependence on the bond strength. It has been shown elsewhere [5] that the ratio $U/D = F_b/F_b'$ where F_b and F_b' are the force and slope respectively of the force-elongation curve evaluated at the middle of the associated force drop. The quantity F_b/F_b' has the dimensions of length. It would seem then that if the ratio U/D (or F_b/F_b') were averaged over a number of bond breaks a suitable parameter for characterizing bond strength would be obtained. This parameter could be thought of as a length characteristic of the average elongation required to break a bond.

When attempts were made to evaluate this parameter it was found that the quantities F_b/F_b' measured tended to have a constant value independent of the force level at break F_b. In several experiments to check this a specimen was elongated, the elongation removed, and the specimen reelongated in order to obtain a complete force-elongation curve to break. Corresponding values of F and F' were measured at various points along this curve. For all specimens tested these points were found to satisfy the linear relationship $F' = a + bF$, where a and b are constants. At large values of F the quantity F'/F is essentially constant. Thus it can be inferred that the parameter just proposed is insensitive to bond strength. However the universal relationship between F' and F is a valuable discovery, because from it the force-elongation equation for the specimen can be derived, and the constants in this equation can be used to characterize the network.

In concept, perhaps the simplest and most direct way to characterize interfiber bond strengths is to measure the force levels at which bonds break, adjust these force levels to compensate for deterioration in the network, and then calculate a parameter from these adjusted force levels. This can be done in several different ways. In order to perform the calculations, however, it is necessary to know the constants characterizing the force-elongation behavior of the specimen network. These force level parameters, therefore, will be developed at the end of this report, after the force-elongation behavior of the network has been discussed.

Materials Studied and Experimental Procedures.

During the past several years a number of low-density open-web handsheets had been prepared from Northern and Southern Kraft woodpulps and from unfractionated and fractionated currency pulps. A description of these pulps and the methods used for preparing the handsheets has been given elsewhere [4]. Force-elongation curves were obtained for a series of Northern Kraft handsheets having densities varying from 1.5 to 3.5 g/m^2, and for a series of Southern Kraft handsheets having densities varying between 1.75 and 3.5 g/m^2. Force-elongation data were also obtained for handsheets prepared from Northern and Southern Kraft woodpulps that had been subjected to different amounts of beating. In addition data were obtained for handsheets made from various currency pulps. A description of the handsheets tested is given in tables 1 and 2.

Tensile tests were conducted on specimens 2 cm long and 1 cm wide except for one series in which the specimen dimensions were varied to investigate the effect of specimen shape and size. Crosshead speed of the tensile tester was 0.2 cm/min except for the specimens of 5-cm length, which were strained at a speed of 0.5 cm/min. During the tests the direction of extension was frequently reversed to unload the specimen and then resumed to obtain a series of force-elongation curves. Most of these tests had been performed previously, but during the current year data from these tests were reworked and reanalyzed. A few tests were also performed during the current year. In these tests more force-elongation curves were obtained for a given specimen in order to provide better data on progressive deterioration in the specimen during a test.

Tests were performed in a laboratory maintained at 20° C, 50% R.H. on specimens stored under these conditions at least 12 hours before testing. The handsheets from which the specimens were obtained were not always stored under these controlled conditions, and may have changed somewhat during the time interval between preparation and testing.

Characterization of the Force-Elongation Curve.

In a previous report [4] it was shown that the force-elongation curve of a test specimen usually can be fit by the equation

$$F = C_1 e^{x/x_c} - C_2 \qquad (1)$$

where F is the tensile force sustained by the specimen, and x is a distance variable measured in the direction of the elongation. C_1, C_2 and x_c are constants evaluated from the data. In this section the method of fitting force-elongation data is reviewed, and the ideas involved are further clarified and developed.

Consider the recorder trace, figure 2, obtained by testing a specimen from a 2.5 - g/m^2 handsheet of Northern Kraft woodpulp (29-119-1, specimen 3). The specimen was 2 cm long by 1 cm wide. 1 cm of chart travel corresponds to a specimen extension of 0.01 cm, and full scale force for the specimen was 98 mN (10 g). At intervals the test was stopped, the extension was reversed until the tensile force in the specimen was zero, and the specimen then reextended to obtain a new force-elongation curve. Recording traces during reversal of the crosshead travel are not shown.

The general condition of the specimen at various stages is noted on the figure. For the first five reextensions the specimen did not develop any large holes or tears, and the state of the specimen at the beginning of the reextension is described as "intact". At later stages of the test the specimen developed holes and tore at the edges as it deteriorated.

In figure 3 are shown plots of F', or force per unit elongation, versus force F for the first eight reextended force-elongation curves of figure 2. The origin of the F' scale has been shifted vertically for each of these plots, and arbitrary units have been selected to display the plots to best advantage. The curve corresponding to each plot is identified by its region of chart travel. The plots are seen to be linear and to have positive intercepts on the F' axis. Plots similar to these have resulted from most of the tests on specimens formed from woodpulps.

It will be noticed that the early points corresponding to small values of F usually fall below the straight line. At these beginning force values the slope of the force-elongation curve increases rapidly as the network fibers start to orient. It is only after the initial orientation that the network behaves so that

F' increases linearly with F. Also it has been found in general that F',F data from the initial force-elongation curve in a test does not fit well to a straight line. In this instance F' usually rises rapidly, becomes constant until F is moderately large, and then increases linearly with F. Data from the first two reextension curves often tend to plot in similar fashion. This behavior is thought to arise from an initial maldistribution of forces in a network, and will be discussed more completely later. It is only after the network has been "broken in" that F' has a good linear correspondence with F. In the plot of figure 3 data from the initial force-elongation curve therefore has been omitted.

Sometimes when a test specimen is unloaded the recorder trace does not return to the original zero force level. This behavior is usually an effect of hysteresis. Its consequences can be minimized by waiting a few minutes before reextending the specimen, but it is more practical to reextend immediately and to correct data from the new curve for the change in baseline. Sometimes a shift of baseline is due to a change in the recorder system, a circumstance especially likely because of the high sensitivities required by these tests. Thus a correction for baseline shift should be made in any event.

Force-elongation curves of these paper network specimens exhibit viscoelastic, or hysteretic, behavior. Thus the slopes of the force-elongation curves will depend upon the rate of extension. The rate of extension therefore must be maintained at some standard value in all of the tests in order to obtain comparable results. This precaution should minimize the influence of viscoelasticity, but the possibility of other hysteretic effects should be kept in mind in the interpretation of data.

Let the equation of a straight line faired through a plot such as those shown in figure 3 be

$$\frac{dF}{dx} = \frac{1}{x_c}(C_2 + F) \tag{2}$$

x_c is the reciprocal of the slope of the straight line, and C_2/x_c is the intercept on the F' axis. Eq (1), given previously, is the solution of eq (2). In order to find the constants C_2 and x_c, eq (2) is fit by the least-squares technique to F',F data from the force-elongation curve. The constant C_1 is then found by fitting eq (1) (with C_2 and x_c already evaluated) to x,F data from the force-elongation curve.

According to eq (1), F becomes zero when

$$x = x_o = \ln(C_2/C_1) \tag{3}$$

Thus eq (1) can be put in the form

$$F = C_2[e^{(x-x_o)/x_c} - 1] \tag{4}$$

x_o is the value of the distance variable at which the force just increases from zero, so $x - x_o$ is the elongation of the network. As the specimen fibers become better alined, and as the specimen deteriorates with increasing stretch, the unstrained length of the specimen increases. Thus for each successive reextension curve of figure 2, successively increasing values of x_o should be expected.

In figure 4 data from the first eight reextension curves of figure 2 have been corrected for small shifts in the baseline and replotted as solid lines. The constants C_2, x_c and x_o were then determined for these curves, and values of x and F calculated for the fitting curve from eq (4). The fitting curves, plotted as dashed lines, are seen to be in good agreement with the original data in this instance. Only the values of x_o, calculated from eq (3), differ slightly from the values might have been determined by inspection of the original data.

When a fibrous network is stretched segments of the fibers composing the network tend to orient themselves in the direction of the extension through the combined action of forces along their lengths. As the extension proceeds more and more of these segments between bonds become oriented and bear load. As a result of this process the force-elongation curve has a slope that increases with increasing elongation. Thus the reaction of the network to extension could be modeled by a system of parallel filaments of unequal length, each filament adding to the resistive force of the system as the extension is increased. This model, discussed in a previous report [4], provides an insight into the meaning of the constants C_2 and x_c.

From an inspection of eq (4) it is seen that x_c has dimensions of length. If the magnitude of x_c is small the force-elongation curve will rise rapidly as $x - x_o$ increases. This is the kind of behavior that might be expected to result from testing a network composed of many meshes of small average size. The small mesh size assures that additional parallel pathways that sustain force are rapidly formed during extension of the network. This suggests that x_c is a parameter having a characteristic length that is proportional to the average

mesh size of the network. If this assumption is valid x_c might be of use in finding values for the force-drop and energy-loss parameters, and might provide an additional indication of the number of bonded crossovers per unit area in a test specimen.

Eq (4) can be rearranged in the form

$$F = C_2\xi/x_c + C_2[e^{\xi/x_c} - 1 - \xi/x_c] \qquad (5)$$

where ξ is the elongation $x - x_o$. The linear term in eq (5) represents the contribution of the parallel pathways that sustain force immediately upon stretching. The nonlinear term represents the contribution of additional pathways that form as the elongation is increased. The value of each term is proportional to C_2. Thus C_2 may be expected to increase as the width of the specimen is increased, or as the density in g/m^2 of specimens made from the same pulp furnish is increased.

During a test the specimen deteriorates, and as a result its unstrained length increases to a value given by

$$\ell = \ell_o + x_o - x_s \qquad (6)$$

where ℓ_o is the original unstrained length of the specimen, and x_s is the value of x_o for the initial force-elongation curve. For figure 2, the test specimen had an initial unstrained length of 2 cm, and the value of x_s as read from the recording trace was 0.0 cm. The quantity x_c has been termed a characteristic length, but in actuality this expression is strictly appropriate only when specimens characterized by this term have the same unstrained length. During a test the unstrained length ℓ of the specimen increases slightly. Therefore in comparing results for the different reextension curves values of the characterisitc strain x_c/ℓ should be used. The quantity C_2 has the dimensions of force and should not be explicitly dependent on the length ℓ.

Table 3 gives the values of constants characterizing the force-elongation curves of figure 2. According to this table the constant C_2 decreases rapidly in value as the specimen deteriorates during a test. The nature of this decrease is demonstrated in figure 5 where C_2 is plotted logarithmically versus the fractional increase in length $\ell/\ell_o - 1$. The straight line approximating this dependence is passed through the data points determined from the third, fourth, fifth and sixth reextension curves. The first two points were too high, indicating that the specimen probably was not "broken in" by the initial extension. The points

corresponding to the seventh and eighth reextension curves are also high. According to the notes on figure 3 the specimen was visually intact for the first five reextensions and deterioration was noticible starting with the sixth reextension. This increase in the value of C_2 after deterioration has become noticible was usually observed in the other specimens tested. A mechanism accounting for this increase is not known, however it seems reasonable to assume that deterioration at this stage is so extensive that the shape of the specimen has become changed. Values of the constants obtained with this reshaped specimen should no longer be compared with values applicable during the early stages of deterioration.

A value for ln C_2 characteristic of the undeteriorated but broken-in specimen can be obtained by extrapolating along the approximating straight line to the point where the fractional increase in length $1 - \ell/\ell_o$ is zero. For the specimen of figure 3 this extrapolated value $(\ln C_2)_o$ is -2.19. The corresponding value of $(C_2)_o$ is 0.1119 N. As is evident in this example however, considerable subjective judgement is involved. In many cases the plotted data show more scatter, possibly due to different kinds and locations of damage incurred by a specimen as the tensile test progresses. In these situations the extrapolated value of C_2 and what this value represents become even more uncertain.

In figure 6 characteristic strain data x_c/ℓ obtained from the test of figure 2 are plotted versus the fractional increase in specimen length ℓ/ℓ_o -1. The characteristic strain seems to decrease linearly with fractional increase in specimen length, according to data from the third through sixth reextension curves. Characteristic strains from the first two reextension curves are too high, possibly indicating that the specimen has not yet been broken in. The decrease of characteristic strain with increasing deterioration of the test specimen was usually observed in the other tests but the reason for this decrease is not understood and requires further study. The points corresponding to the seventh and eighth reextension curves plot high, possibly indicating that the specimen by then is so deteriorated that its shape and other characteristics have changed.

A value for x_c/ℓ characteristic of the undeteriorated but broken-in specimen can be obtained by extrapolating along the approximating straight line to the point where the fractional increase in length is zero. For the specimen of figure 3 this extrapolated value $(x_c/\ell)_o$ is 0.0225. the corresponding value of $(x_c)_o$ is 4.50×10^{-4} m.

Values of $(C_2)_o$ and $(x_c)_o$ found by extrapolation are parameters characteristic of the force-elongation behaviors of the specimens tested. If these parameters are evaluated for specimens cut from the same handsheet, scatter in the values obtained should be indicative of the textural homogeneity of the handsheet. The quantities $(C_2)_o$ and $(x_c)_o$ may also be of use in reducing bonding parameters evaluated from various specimens or various handsheets to values corresponding to standard values of $(C_2)_o$ and $(x_c)_o$. Standard values of the bonding parameters thus obtained then could be intercompared.

Results for Woodpulp Specimens.

A short series of tests was made to see if the values of C_2 and x_c depended upon the shape of the specimen. For these tests specimens of cm-width x length 1 x 2, 1 x 5 and 2 x 5 were cut from the same handsheet. The handsheet (29-119-1,2,3) was of 2.5 g/m^2 density formed from Northern Kraft woodpulp beaten 5,000 revolutions in a laboratory beater. In the tests crosshead speed was 0.2 cm/min for the specimens of 2-cm length and 0.5 cm/min for the specimens of 5-cm length. Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ obtained from these tests are given in tables 4, 5 and 6.

Scatter in the values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ for the 1 x 2-cm specimens, table 4, was too large for any meaningful comparisons with results for the other two specimen sizes. Scatter in the values for the 2 x 5 and 1 x 5-cm specimens, according to the standard deviations found, was appreciably lower. This agrees with expectations, as the areas of these two specimen sizes are 10 cm^2 and 5 cm^2 respectively, but the area of the 1 x 2-cm specimen is only 2 cm^2. Thus the effects of textural inhomogeneities in the specimens of larger area evidently are smoothed out.

For the 2 x 5-cm specimens the average value of $(\ln C_2)_o$ is -2.19 corresponding to a C_2 value of 0.1119 N. For the 1 x 5-cm specimens $(\ln C_2)_o$ is -2.98 corresponding to a C_2 value of 0.0508 N. The C_2 value for the 2 x 5-cm specimens is roughly twice that for the 1 x 5 cm-specimens, in agreement with expectations. The values of $(x_c/\ell)_o$ for the 2 x 5 and 1 x 5 specimens are approximately the same, indicating that the value of x_c is not dependent on the width of the specimen. This observation is consistent with the idea that x_c is a length characteristic of the average mesh size of the specimen network.

The tests just described indicate that specimen dimensions of 2 x 5 or 1 x 5 cm yield the most precise results. The 1 x 5-cm specimen size might be preferable, however, because force drops that occur during a test are more clearly delineated. In addition the long narrow configuration tends to minimize edge effects at the clamped ends, which might prevent proper orientation of the network fibers and an even sharing of the tensile force throughout the width of the specimen.

It should be noted that these tests, performed on the handsheet designated as 29-119-1,2,3 were performed in April 1981, and featured more frequent reextensions of the specimens than had been done in tests performed previously

in 1980. This tended to improve the precision. Other results to be presented will not be as precise as they were mostly obtained from tests on 1 x 2-cm specimens in which fewer reextensions were performed.

According to previous discussion it is expected that C_2 should increase as the density of the test specimen is increased, because the higher density specimens have more fibers per unit area and hence more pathways to sustain a tensile force. It is also expected that x_c should decrease as the density of the test specimen is increased, because the higher density specimens have a smaller mesh size. In order to verify these expectations values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ were obtained for a series of Northern and a series of Southern Kraft woodpulp handsheets of various densities. The results are given in tables 7 and 8. The fluctuations in the values $(\ln C_2)_o$ and $(x_c/\ell)_o$ as indicated by the standard deviations are large, and it is hard to discover any dependence of these quantities upon handsheet density. This dependence is more obvious in the plots of figures 7 through 10, where it is seen that $(\ln C_2)_o$ increases and $(x_c/\ell)_o$ tends to decrease with increase in handsheet density, in agreement with expectations. Linear regression lines have been drawn through the data points to make this more obvious.

The fibers in the Southern pulp furnish are coarser than the fibers in the Northern pulp furnish. Thus a handsheet formed from the Southern pulp would have fewer fibers per unit area than would the Northern pulp handsheet of the same density. There would be fewer oriented fiber pathways in an extended Southern pulp specimen, hance C_2 for the Southern pulp would be less than C_2 for a Northern pulp specimen of the same density. Also the mesh size of the Southern pulp specimen would be larger, hence x_c for the Southern pulp would be larger than x_c for a Northern pulp specimen of the same density. It is interesting to note in tables 7 and 8 that for handsheets of Northern and of Southern pulp of densities 1.75, 2.0, 2.5 and 3.5 g/m^2, C_2 for the Southern pulp is less than C_2 for the Northern pulp (except for Northern pulp handsheet 29-1-4), and x_c for the Southern pulp is larger than x_c for the Northern pulp. The imprecision of the measurements, however, is so great that this apparent confirmation of expectations must be regarded as somewhat fortuitous.

The parameters $(\ln C_2)_o$ and $(x_c/\ell)_o$ were determined for a series of handsheets of the same density prepared from Northern Kraft woodpulp beaten various amounts in a laboratory beater. Parameters were also obtained for a

similar series of handsheets prepared from Southern Kraft woodpulp. The results are given in tables 9 and 10. The data from these tables are plotted in figures 11 through 14. The effect of beating should be to increase the number of bonds per unit area of the handsheet. Thus the number of pathways sustaining tensile force should increase, and the mesh size should decrease with the degree of beating. Inspection of table 9 and figures 11 and 12 shows that these expectations are borne out for the Northern Kraft pulp handsheets. This dependence of $(\ln C_2)_o$ and $(x_c/\ell)_o$ upon beating is not so pronounced for Southern pulp. The linear regression lines plotted in figures 13 and 14, however, show the expected trend.

Results for Currency pulp Specimens

The methods described previously for evaluating the parameters x_c, x_o, C_1 and C_2 have worked well for test specimens made from Northern and Southern Kraft woodpulps, but when specimens made from currency pulps are tested, a difficulty arises. To illustrate, consider the graphical data of figures 15 and 16. The force-elongation curve, figure 15, was obtained by testing a specimen (28-159-1, specimen 12) from a 2.5-g/m^2 handsheet of unfractionated currency pulp. Figure 16 gives plots of slope F' versus force F for the first seven reextended force-elongation curves of figure 15. The origin of the F' scale has been shifted vertically for each of these plots, and arbitrary units have been selected to display the plots to best advantage. The curve in figure 15 corresponding to each plot is identified by its region of chart travel.

The F, F' data for the reextension curves can be fit by a straight line, as required by eq (2), but only for forces exceeding approximately 1.5 units in the plots of figure 16. In the initial regions the F' data fall above the fitted straight lines. The intercepts on the F' axes, which should be equal to the values of C_2/x_c, are too low for the first three curves, and are negative for the later curves.

The example, figures 15 and 16, is typical not only for tests on handsheet specimens of unfractionated currency pulp but also for the specimens made from various length fractions and from a pulp from which the fines were removed. The F, F' behavior just shown can be explained if it is assumed that when a currency pulp specimen is stretched there are two mechanisms operating in parallel: a strong spring-like force that dominates the initial stages of the extension, and a force mechanism described by eqs (1-4) that dominates the later stages of the extension. The strong spring-like force might simulate the effort required to orient a fibrous network partially immobilized by numerous adhesions between small fibrils and by the cementing action of fines. The initial strong spring-like force might also result from an initial maldistribution of forces in which the force resisting extension is sustained by only a few highly extended fiber pathways. The force-elongation behavior of such highly extended pathways might be nonlinear at very small extensions but rapidly approaches linearity while the specimen extension is still small.

The behavior just described can be approximated by the equation:

$$F = k(x - x_o) + C_1 e^{x/x_c} - C_2$$

$$= k(x - x_o) + C_2[e^{(x-x_o)/x_c} - 1]$$

$$= kx + C_1 e^{x/x_c} - C_3 \tag{7}$$

Here a linear force of spring constant k has been added to eq (1), and it is assumed that the force in this spring and the forces in the mechanism acting in parallel are both zero at extension x_o. The parameter C_3 is a constant defined by

$$C_3 = C_2 + kx_c \ln (C_2/C_1) \tag{8}$$

The parameters k, x_c, C_1, C_2 and x_o in eq (7) can be evaluated by the following proceedure: The first order differential equation corresponding to eq (7) is

$$F' - k = \frac{1}{x_c} [C_3 + (F - kx)] \tag{9}$$

If a value is assumed for k, the parameters C_3 and x_c can be found by fitting eq (9) to experimental x, F, F' data. Parameter C_1 then can be found by fitting eq (7) (with k, x_c, C_3 known) to experimental x, F data. Parameter C_2 is found from eq (8) by Newton's method, and parameter x_o is then found from eq (3). The values of the parameters just found depend upon the value of k that was assumed. to find the proper value of k it is necessary that the quantity

$$\sum_{i=1}^{n}[F_i' - kx_i - C_1(k)e^{x_i/x_c(k)} + C_3(k)]^2 \text{ be a minimum.}$$

This quantity can be minimized with respect to k by a systematic trial and error process.

Force-elongation curves were obtained on specimens made from handsheets of currency pulp, of several length fractions of the pulp, and of a pulp from which the fines had been removed. The test procedures have been described previously. Full-scale forces encountered during the tests were usually 100 mN (10g) or less. The parameters characterizing the first few curves for each specimen are given in tables 11 through 15.

It is seen from these tables that in almost every instance k is greater than zero; i.e., a springlike force is required to provide a good fit of eq (7) to

force-elongation data. Data in table 11 characterize specimens made from unfractionated pulp (28-159-1). Data in tables 12, 13, 14 and 15 characterize length fractions (28-159-2,3,4) and a pulp from which the fines were removed (28-159-5), and yet in these tables the values of k are about the same as those in in table 11. This suggests that if the initial strong springlike force measures the effort required to orient a partially immobilized network, this immobilization may be attributed more to numerous adhesions between microfibrils than to the cementing action of fines. Adhesion between microfibrils could also produce inhomogeneities in the network and cause an initial maldistribution of tensile forces resisting extension. The cementing action of fines and the presence of numerous adhesions involving microfibrils are evident in pictures taken with an scanning electron microscope [4]. The initial strong springlike force is usually not present when unfibrillated and fine-free specimens made from Kraft woodpulp are tested.

The parameter x_c, which is related to the average mesh size of the specimen network, has a smaller value for specimens made from the short-length fraction of currency pulp, table 14, than for the other specimens, as might be expected. According to tests on woodpulp specimens, x_c remains constant or decreases slightly for successive reextension curves. This trend can also be observed in the currency pulp data. However there is also a tendency, in tests on the same specimen, for low values of x_c to be associated with high values of k, and vice versa. In tests on woodpulp specimens, where k is usually zero, fluctuations in x_c are not so prevalent. It is likely that in tests on currency pulp specimens the values of k cannot be determined very accurately, and that the constant fluctuation of k values produces increased variability in values of x_c.

The parameter C_2 characterizes the strength of the nonlinear network force in a fashion analogous to the way k characterizes the strength of the springlike force. Previous tests on woodpulp specimens indicate that in successive force-elongation curves for the same specimen C_2 decreases rapidly, thus providing an index sensitive to deterioration of the specimen. This rapid decrease in C_2 for successive force-elongation curves is also apparent in the tabular data, tables 11 through 15. The values of C_2 undergo fluctuations influenced by variations in values of k, but this probably must be accepted. It is necessary to determine some approximate value of k in order to determine a meaningful value of C_2.

Although the fluctuations observed in k and C_2 limit their use in characterizing currency pulps, the measurements demonstrate the presence of an initial strong springlike force, and suggest that much of the binding between fibers is achieved through a large number of small bonds to microfibrils. In such a circumstance it would not be appropriate to attempt a characterization of interfiber bond strength in currency pulps through the use of force drop or energy loss parameters. These bonding parameters are more applicable to Kraft woodpulps.

Force Level Parameters for Characterizing Interfiber Bonding

In concept, perhaps the simplest and most direct way to characterize interfiber bonding is to measure the average force sustained by the specimen when bond breaks occur. However a crude parameter obtained by averaging the breaking forces over a specified extension would not provide a very precise characterization of interfiber bonding. The force sustained by the specimen when bonds are breaking depends upon the specimen deterioration . The force also depends upon how the component forces are distributed among the fiber segments. In the early stages of an extension the force is channeled through only a few fiber pathways, and bonds along these pathways must be broken in order to obtain a force distribution sufficiently uniform for characterization purposes. Moreover, the distribution of fibers in each test specimen is nonuniform, and various flaws such as small holes are present. Thus at various stages of an extension the fiber segments sustaining most of the force may be expected to shift from one region of the specimen to another.

In order to compensate for these effects it is necessary to know the values of the constants C_2, x_c and x_o that characterize the force-elongation behavior of the specimen, and to know how these constants change as the tensile test progresses. A more suitable force level parameter then might be determined as follows: From the values of C_2 and x_c characterizing a series of reextension force-elongation curves obtained during a test, select a standard pair of values C_{2s} and x_{cs}. Measure the breaking forces in the vicinity of each reextension curve, and then calculate what the equivalent breaking force would be for a specimen having a force-elongation behavior governed by the coefficients C_{2s} and x_{cs}. Average these equivalent breaking forces to obtain the force level parameter.

A more detailed procedure will now be discussed:

1. Perform a tensile test upon a specimen for which many reextension force-elongation curves are obtained, and evaluate the constants of these curves. For the i th curve let the constants be designated as C_{2i}, x_{ci} and x_{oi}.

2. Assume that in a certain region of force drops the force-elongation behavior is described by the i th curve; measure the average force level in this region by integrating under the curve of breaking forces and dividing by the amount of extension. Designate this average breaking force as F_{bi}. Substitute F_{bi} into eq (4)

$$F_{bi} = C_{2i}[e^{(x_{bi}-x_{oi})/x_{ci}} - 1] \tag{10}$$

and solve for $\xi_{bi} = x_{bi} - x_{oi}$, obtaining

$$\xi_{bi} = x_{ci} \ln\left(1 + \frac{F_{bi}}{C_{2i}}\right) \tag{11}$$

3. Find the unstrained length ℓ_i of the specimen from eq (6) and then calculate

$$\epsilon_{bi} = \frac{\xi_{bi}}{\ell_i} \tag{12}$$

The quantity ϵ_{bi} is the average strain required to produce a bond break in the region of the i th curve.

Ideally the ϵ_{bi} for a given specimen should all have approximately the same value. Some tentative calculations, however, have shown that the ϵ_{bi} increase with increasing i and approach a constant value. Thus to find a suitable average $\overline{\epsilon_b}$ for the specimen the first ϵ_{bi} used in the averaging should not be less than approximately 5% of the average value. The value of $\overline{\epsilon_b}$ should be a weighted average; that is, each ϵ_{bi} should be multiplied by the range of extension to which it applies, and the sum of these products should be divided by the sum of the extension ranges. The parameter $\overline{\epsilon_b}$ thus obtained characterizes the average breaking strain for a specimen whose force-strain parameters C_{2i} and ϵ_{ci} lie within a certain range of experimental values.

To obtain a force level parameter for the specimen:

1. Substitute the F_{bi} into eq (4) and solve for

$$e^{\epsilon_{bi}/\epsilon_{ci}} = 1 + \frac{F_{bi}}{C_{2i}} \tag{13}$$

Calculate only those values of $e^{\epsilon_{bi}/\epsilon_{ci}}$ for which the ϵ_{bi} have approximately the same value, as determined in the previous calculation of $\overline{\epsilon_b}$. Calculate also the quantities $\epsilon_{ci} = x_{ci}/\ell_i$.

2. Choose a characteristic strain ϵ_{cs} such as , for instance, the value of ϵ_{ci} for which i has the lowest value in the series of $e^{\epsilon_{bi}/\epsilon_{ci}}$ that was just calculated. Calculate the quantities

$$e^{\epsilon_{bi}/\epsilon_{cs}} = [e^{\epsilon_{bi}/\epsilon_{ci}}]^{\epsilon_{ci}/\epsilon_{cs}} \tag{14}$$

The values of $e^{\epsilon_{bi}/\epsilon_{cs}}$ should be approximately the same because the values of ϵ_{bi} are approximately the same.

3. Calculate the quantities

$$F_{sbi} = C_{2s}[e^{\epsilon_{bi}/\epsilon_{cs}} - 1] \tag{15}$$

using for C_{2s} the value of C_{2i} corresponding to the ϵ_{ci} chosen for ϵ_{cs}. The parameter $\overline{F_{sb}}$, obtained by a weighted average similar to that for $\overline{\epsilon_b}$, gives the breaking force level for the specimen corresponding to the force-strain coefficients C_{2s} and ϵ_{cs}.

A parameter $\overline{F_{sb}}^*$ obtained by a simpler calculation could be used alternatively. This parameter is defined as

$$\overline{F_{sb}}^* = C_{2s}[e^{\overline{\epsilon_b}/\epsilon_{cs}} - 1] \tag{16}$$

The parameter $\overline{F_{sb}}$ gives the average force level for bond breaks in a specimen having a force-strain relationship governed by the constants C_{2s} and ϵ_{cs}. In order to find a parameter $\overline{\overline{F_{sb}}}$ characterizing a number of specimens from the same handsheet sample, it is necessary to choose arbitrary values of $\overline{C_{2s}}$ and $\overline{\epsilon_{cs}}$ for the sample, reduce each $\overline{F_{sb}}$ to its equivalent value corresponding to $\overline{C_{2s}}$ and $\overline{\epsilon_{cs}}$, and then average these values. When this parameter $\overline{\overline{F_{sb}}}$ is used for specification purposes The values of $\overline{C_{2s}}$ and $\overline{\epsilon_{cs}}$ upon which it depends must also be given.

A corresponding parameter $\overline{\overline{\epsilon_b}}$ for a handsheet sample might be obtained by averaging together the $\overline{\epsilon_b}$ for a number of specimens. This parameter might be independent of the values of C_{2s} and ϵ_{cs} for the individual specimens, and of $\overline{C_{2s}}$ and $\overline{\epsilon_{cs}}$ for the sample, but this concept has not been tested experimentally.

It has been noted previously that the concept of the average bond strength of an interfiber junction is valid in the case of papers made from woodpulp. For papers made from currency pulp the fibers are joined together by many bonds between microfibrils and rigidly held by the gluing action of fines. In this case characterization of an interfiber bond strength by such parameters as average force drop or energy loss per break is no longer appropriate. For both kinds of paper, however, it might be suitable to characterize the overall ability to hold together by means of a force level parameter. In the preceding discussion this parameter was developed for woodpulp papers, using eq (4). Similar more general parameters applicable to both currency and woodpulp paper might be developed by applying the same rationale to eq (7).

As noted above the force level and elongation parameters must be considered as tentative. There has not been sufficient time during this reporting period to test their validity and usefulness. The rationale, however, seems plausible and worthy of application at some future time.

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Table 1.

Description of Woodpulp Handsheets Tested

Handsheet description	Pulp	Density g/m^2	Beating rev.	Preparation pressure kPa	Preparation date	Test date
28-159-6	Northern	2.5	5,000	353	10/20/76	2/19/80
28-159-7	Southern	2.5	5,000	44	6/30/76	2/28/80
28-159-8	Southern	2.5	1,000	44	6/21/76	2/20/80
29-159-9	Southern	2.5	2,000	44	6/23/76	2/26/80
28-159-11	Northern	1.5	5,000	353	5/3/77	2/13/80
28-159-12	Northern	1.75	5,000	353	5/3/77	2/14/80
29-1-1	Northern	2.0	5,000	353	5/3/77	2/15/80
29-1-2	Northern	2.5	1,000	44	6/21/76	2/20/80
29-1-3	Northern	2.5	2,000	44	6/23/76	2/21/80
29-1-4	Northern	2.5	5,000	44	6/25/76	2/21/80
29-1-5	Northern	2.5	10,000	353	8/11/76	2/22/80
29-1-6	Southern	2.5	10,000	662	4/15/76	2/28/80

(Continued)

Table 1 Continued.

Handsheet designation	Pulp	Density g/m^2	Beating rev.	Preparation pressure kPa	Preparation date	Test date
29-118-1	Northern	2.25	5,000	345	7/8/80	3/31/81
29-118-3	Northern	3.0	5,000	345	7/9/80	3/31/81
29-118-5	Northern	3.5	5,000	345	7/9/80	3/31/81
29-118-7	Southern	1.75	5,000	345	7/15/80	4/1/81
29-118-10	Southern	2.0	5,000	345	7/14/80	4/1/81
29-118-12	Southern	3.5	5,000	345	7/14/80	4/2/81
29-119-1 1/	Northern	2.5	5,000	353	10/20/76	4/15/81
29-119-2 2/	Northern	2.5	5,000	353	10/20/76	4/14/81
29-119-3 3/	Northern	2.5	5,000	353	10/20/76	4/19/81

1. Same handsheet as 28-159-6 but different test date; specimen length 2 cm, width 1 cm.
2. Same handsheet as 28-159-6 but different test date; specimen length 5 cm, width 1 cm.
3. Same handsheet as 28-159-6 but different test date; specimen length 5 cm, width 2 cm.

Table 2.

Description of Currency Pulp Handsheets Tested[1/].

Handsheet designation	Fraction	Fractional[2/] yield	Preparation date	Test date
28-159-1	Unfractionated	1.00	1/30/79	1/17/80
28-159-2	Fraction I (#14 mesh)	.36	2/2/79	1/18/80
28-159-3	Fraction II (#35 mesh)	.25	2/2/79	1/21/80
28-159-4	Fraction III (#65 mesh)	.11	2/5/79	1/22/80
	Fraction IV (#150 mesh)	.06		
	Fines	.22		
28-159-5	Fractions I, II, III, IV recombined	.78	2/13/79	1/29/80

1. Beaten currency stock was fractionated to yield four fractions. Handsheets from these pulps were of density 2.5 g/m^2, and were prepared at 345 kPa pressure.
2. Weight of dry pulp in the fraction divided by dry weight of unfractionated pulp.

Table 3.

Values of Constants C_2, x_c and x_o for Curves of Figure 2.

Curve	x_o m	ℓ m	ℓ/ℓ_o-1	x_c m	C_2 N	x_c/ℓ	ln C_2
1.2 - 2.0	.12363-3	.0201236	.0061815	.77238-3	.13326+0	.038382	-2.01545
2.0 - 5.6	.20140-3	.0202014	.0100700	.47635-3	.55444-1	.023580	-2.89238
2.8 - 7.2	.29381-3	.0202938	.0146905	.35763-3	.26909-1	.017623	-3.61529
4.0 - 9.2	.41540-3	.0204154	.0207700	.29304-3	.14648-1	.014354	-4.22345
5.6 - 10.4	.54082-3	.0205408	.0270410	.25553-3	.77632-2	.012440	-4.85836
7.2 - 12.0	.67841-3	.0206784	.0339205	.21586-3	.41462-2	.010439	-5.48556
9.2 - 13.6	.89463-3	.0208946	.0447315	.02040-3	.44791-2	.009974	-5.40833
11.2 - 15.6	.11027-2	.0211027	.0551350	.18903-3	.36176-2	.008958	-5.62194

Table 4.

Extrapolated Values $(\ln C_2)_o$ and $(x_c/\ell)_o$ for Specimens[1/] of 1-cm Width and 2-cm Length.

Specimen	$(\ln C_2)_o$ [2/]	Slope[3/]	SD[4/]	n[5/]
29-119-1-3	-2.19	-97.58	.02	4
-4	-3.47	-21.35	.10	6
-5	-2.02	-94.05	.12	5
-6	-2.08	-63.28	.09	4
-12	-2.75	-34.55	.13	7
	-2.50 ± 0.61 std. dev.			

Specimen	$(x_c/\ell)_o$	Slope	SD	n
29-119-1-3	.0225	-.3655	.0006	4
-4	.0128	-.0208	.0005	6
-5	.0260	-.4378	.0009	5
-6	.0248	-.3248	.0012	4
-12	.0217	-.1332	.0009	7
	.0216 ± .0052 std. dev.			

1. Cut from Northern pulp handsheet, 2.5 g/m^2 density.
2. ln is logarithm to the base e. C_2 is expressed in Newtons.
3. Slope of linear regression line.
4. Standard deviation of fit.
5. Number of data points.

Table 5.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ for Specimens[1/] of 1-cm Width and 5-cm Length.

Specimen	$(\ln C_2)_o$ [2/]	Slope[3/]	SD[4/]	n[5/]
29-119-2-4	-3.09	-29.50	.16	7
-6	-3.19	-80.60	.10	4
-7	-2.78	-94.63	.15	4
-10	-2.87	-58.21	.02	4
-12	-2.95	-88.54	.08	4
	-2.98 ± 0.17 std. dev.			

Specimen	$(x_c/\ell)_o$	Slope	SD	n
29-119-2-4	.0237	-.2300	.0010	7
-6	.0184	-.2726	.0003	4
-7	.0243	-.5521	.0012	4
-10	.0171	-.1807	.0001	4
-12	.0183	-.1750	.0014	4
	.0204 ± .0034 std. dev.			

1. Cut from Northern pulp handsheet, 2.5 g/m^2 density.
2. ln is logarithm to the base e. C_2 is expressed in Newtons.
3. Slope of linear regression line.
4. Standard deviation of fit.
5. Number of data points.

Table 6.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ for Specimens[1/] of 2-cm Width and 5-cm Length.

Specimen	$(\ln C_2)_o$[2/]	Slope[3/]	SD[4/]	n[5/]
29-119-3-4	-2.40	-97.03	.14	5
-6	-2.18	-45.08	.23	6
-7	-2.23	-44.18	.08	5
-9	-2.18	-35.49	.19	3
-10	-1.94	-114.35	.26	5
	-2.19 ± 0.17 std. dev.			

Specimen	$(x_c/\ell)_o$	Slope	SD	n
29-119-3-4	.0190	-.2838	.0004	5
-6	.0191	-.1495	.0013	6
-7	.0186	-.1428	.0003	5
-9	.0194	-.2257	.0012	3
-10	.0163	-.3284	.0013	5
	.0185 ± .0013 std. dev.			

1. Cut from Northern pulp handsheet, 2.5 g/m^2 density.
2. ln is logarithm to the base e. C_2 is expressed in Newtons.
3. Slope of linear regression line.
4. Standard deviation of fit.
5. Number of data points.

Table 7.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ for Northern Kraft Pulp Handsheets of Various Densities.

Handsheet designation	Density g/m^2	$(\ln C_2)_o$[1]	$(x_c/\ell)_o$
28-159-11	1.5	-5.75 ± 0.34[2]	.0230 ± .0047[2]
28-159-12	1.75	-4.06 ± 0.34	.0217 ± .0068
29-1-1	2.0	-4.53 ± 0.54	.0167 ± .0024
29-118-1	2.25	-3.53 ± 0.49	.0215 ± .0038
28-159-6	2.5	-3.12 ± 0.51	.0185 ± .0050
28-119-1	2.5	-2.50 ± 0.61	.0216 ± .0052
29-1-4	2.5	-3.88 ± 0.74	.0172 ± .0044
29-118-3	3.0	-3.14 ± 0.54	.0198 ± .0033
29-118-5	3.5	-2.27 ± 0.89	.0166 ± .0029

1. ln is logarithm to the base e. C_2 is expressed in Newtons.
2. Standard deviation. Values given are the averages of determinations from 5 specimens in most cases.

Table 8.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$ for Southern Kraft Pulp Handsheets of Various Densities.

Handsheet designation	Density g/m^2	$(\ln C_2)_o$ [1]	$(x_c/\ell)_o$
29-118-7	1.75	-5.10 ± 0.24 [2]	.0273 ± .0018 [2]
29-118-10	2.0	-4.74 ± 0.55 [3]	.0241 ± .0066 [3]
29-159-7	2.5	-3.37 ± 0.76 [4]	.0290 ± .0064 [4]
29-118-12	3.5	-2.69 ± 0.72 [5]	.0198 ± .0046 [5]

1. ln is logarithm to the base e. C_2 is expressed in Newtons.
2. Standard deviation. Average of determinations from 2 specimens.
3. Average from 5 specimens.
4. Average from 4 specimens.
5. Average from 10 specimens.

Table 9.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$
for Handsheets of Northern Kraft Pulp
Beaten Various Amounts.

Handsheet designation	Beating revolutions	$(\ln C_2)_o$ [1/]	$(x_c/\ell)_o$
29-1-2	1,000	-4.47 ± 0.31 [2/]	.0212 ± .0028 [2/]
29-1-3	2,000	-3.82 ± 0.52 [3/]	.0182 ± .0017 [3/]
29-1-4	5,000	-3.88 ± 0.74 [4/]	.0172 ± .0044 [4/]
29-1-5	10,000	-3.15 ± 0.64 [3/]	.0160 ± .0026 [3/]

Table 10.

Extrapolated Values of $(\ln C_2)_o$ and $(x_c/\ell)_o$
for Handsheets of Southern Kraft Pulp
Beaten Various Amounts.

Handsheet designation	Beating revolutions	$(\ln C_2)_o$ [1/]	$(x_c/\ell)_o$
28-159-8	1,000	-4.14 ± 0.68 [5/]	.0290 ± .0052 [5/]
28-159-9	2,000	-4.72 ± 0.74 [3/]	.0251 ± .0062 [3/]
28-159-7	5,000	-3.37 ± 0.76 [6/]	.0290 ± .0064 [6/]
29-1-6	10,000	-4.21 ± 0.55 [6/]	.0218 ± .0060 [6/]

1. ln is logarithm to the base e. C_2 is expressed in Newtons.
2. Standard deviation. Average of determinations from 6 specimens.
3. Average from 7 specimens.
4. Average from 8 specimens.
5. Average from 3 specimens.
6. Average from 4 specimens.

Table 11.

Parameters for Handsheet Specimens Made From Unfractionated Currency Pulp (28-159-1).

Specimen No. and (Curve)	k N/m	x_c m	C_1 N	C_2 N	x_o m
1(1)	11.0	.17950E-3	.24538E-2	.71022E-2	.19077E-3
1(2)	0.0	.20785E-3	.60210E-3	.52331E-2	.44943E-3
1(3)	13.4	.12386E-3	.69801E-6	.33919E-3	.76618E-3
7(1)	48.0	.14955E-3	.79367E-3	.68461E-2	.32225E-3
7(2)	17.1	.15515E-3	.33037E-4	.20488E-2	.64036E-3
7(3)	15.1	.15946E-3	.40923E-5	.10350E-2	.88232E-3
7(4)	8.0	.17077E-3	.79007E-6	.66103E-3	.11492E-2
7(5)	3.8	.18841E-3	.24228E-6	.57717E-3	.14650E-2
8(1)	0.0	.14663E-3	.10991E-2	.35426E-2	.17161E-3
8(2)	22.7	.12584E-3	.29037E-4	.47293E-3	.35114E-3
8(3)	10.6	.17516E-3	.39841E-4	.94042E-3	.55376E-3
8(4)	14.0	.15431E-3	.22499E-5	.37837E-3	.79082E-3
8(5)	1.3	.22948E-3	.65904E-5	.66225E-3	.10579E-2
10(1)	44.8	.14360E-3	.10222E-2	.37019E-2	.18480E-3
10(2)	4.7	.17696E-3	.43533E-3	.32734E-2	.35702E-3
10(3)	10.7	.14762E-3	.77245E-5	.57998E-3	.63745E-3
11(1)	5.7	.17823E-3	.85904E-5	.34420E-3	.65777E-3
11(2)	8.4	.14813E-3	.23560E-6	.14788E-3	.95426E-3
11(3)	6.8	.16879E-3	.13995E-6	.14561E-3	.11726E-2
11(4)	5.8	.18279E-3	.70253E-7	.16734E-3	.14213E-2
11(5)	8.3	.17539E-3	.65180E-8	.28822E-3	.18762E-2

Table 11. Continued.

Specimen No. and (Curve)	k	x_c	C_1	C_2	x_o
	N/m	m	N	N	m
12(1)	9.4	.19211E-3	.24179E-3	.10984E-2	.29075E-3
12(2)	0.0	.23586E-3	.23618E-3	.17234E-2	.46876E-3
12(3)	0.1	.24344E-3	.86008E-4	.12839E-2	.65807E-3
12(4)	5.8	.19162E-3	.24834E-5	.39139E-3	.96962E-3
12(5)	8.3	.17513E-3	.13945E-6	.23125E-3	.12984E-2
12(6)	3.4	.22277E-3	.45700E-6	.37860E-3	.14969E-2
12(7)	6.5	.19604E-3	.20523E-7	.30738E-3	.18848E-2
12(8)	3.5	.23027E-3	.37541E-7	.33589E-3	.20952E-2
12(9)	5.9	.19481E-3	.59532E-9	.22025E-3	.24976E-3

Table 12.

Parameters for Handsheet Specimens Made From Currency Pulp Fraction I (28-159-2).

Specimen No. and (Curve)	k	x_c	C_1	C_2	x_o
	N/m	m	N	N	m
2(1)	24.2	.20738E-3	.55819E-3	.16934E-2	.23015E-3
2(2)	19.7	.17192E-3	.35163E-4	.40765E-3	.42126E-3
2(3)	15.1	.20196E-3	.11039E-4	.32697E-3	.68435E-3
2(4)	8.0	.29771E-3	.34276E-4	.82818E-3	.94815E-3
2(5)	9.1	.25387E-3	.20517E-5	.33147E-3	.12909E-2
5(1)	24.8	.99183E-4	.98388E-4	.57215E-3	.17461E-3
5(2)	4.0	.19083E-3	.20132E-3	.11695E-2	.33574E-3
5(3)	7.8	.10443E-3	.14636E-7	.61410E-4	.87111E-3
8(1)	23.9	.13930E-3	.42940E-4	.21509E-3	.22444E-3
8(2)	16.7	.17651E-3	.17207E-4	.16572E-3	.39979E-3
8(3)	11.7	.20686E-3	.96346E-5	.23403E-3	.65991E-3
8(4)	4.5	.32164E-3	.34382E-4	.55075E-3	.89216E-3
8(5)	4.7	.29389E-3	.28264E-5	.42231E-3	.14714E-2
10(1)	14.4	.18855E-3	.20636E-3	.46340E-3	.15253E-3
10(2)	10.7	.17719E-3	.16314E-4	.14243E-4	.38394E-3
10(3)	7.8	.20794E-3	.61396E-5	.12564E-3	.62769E-3
10(4)	7.3	.24032E-3	.30561E-5	.21510E-5	.10223E-2
10(5)	5.5	.25032E-3	.63961E-6	.14121E-3	.13510E-2
10(6)	4.0	.29168E-3	.59859E-6	.18892E-3	.16785E-2
10(7)	5.2	.27454E-3	.53950E-7	.13301E-3	.21442E-2
14(1)	24.2	.14933E-3	.12596E-3	.45033E-3	.19025E-3
14(2)	10.1	.18557E-3	.35457E-4	.28298E-3	.38545E-3
14(3)	2.6	.24944E-3	.29515E-4	.38305E-3	.63938E-3

Table 13.

Parameters for Handsheet Specimens Made From Currency Pulp Fraction II (28-159-3).

Specimen No. and (Curve)	k	x_c	C_1	C_2	x_o
	N/m	m	N	N	m
3(1)	11.2	.21423E-3	.57560E-3	.21276E-2	.28006E-3
3(2)	7.5	.21601E-3	.73670E-4	.67427E-3	.47825E-3
3(3)	3.7	.24705E-3	.24391E-4	.44923E-3	.71975E-3
7(1)	3.6	.22645E-3	.91134E-3	.17026E-2	.14153E-3
7(2)	0.3	.23933E-3	.16033E-3	.28889E-3	.14092E-3
7(3)	0.6	.31575E-3	.10215E-3	.53825E-3	.82474E-3
7(4)	1.4	.31413E-3	.25804E-4	.71412E-3	.10431E-2
8(1)	13.7	.18549E-3	.14984E-4	.25620E-2	.95368E-3
8(2)	6.1	.21357E-3	.58117E-5	.17009E-2	.12129E-2
8(3)	13.2	.16426E-3	.17274E-7	.32618E-3	.16173E-2
10(1)	21.6	.14877E-3	.63033E-3	.34575E-2	.25321E-3
10(2)	6.1	.14941E-3	.14781E-4	.65930E-3	.56744E-3
10(3)	9.1	.17371E-3	.44125E-5	.20830E-3	.66958E-3
11(1)	1.6	.17947E-3	.15483E-2	.53673E-2	.22311E-3
11(2)	0.0	.19266E-3	.23023E-3	.11048E-2	.30215E-3
11(3)	8.4	.11531E-3	.26113E-6	.35571E-4	.56668E-3

Table 14.
Parameters for Handsheet Specimens Made From
Currency Pulp Fraction III (28-159-4).

Specimen No. and (Curve)	k	x_c	C_1	C_2	x_o
	N/m	m	N	N	m
2(1)	0.0	.12182E-3	.11679E-2	.35463E-2	.13530E-3
2(2)	0.0	.10063E-3	.40726E-4	.19714E-3	.15870E-3
2(3)	17.3	.81302E-4	.82898E-7	.12003E-3	.59170E-3
2(4)	11.6	.81615E-4	.13341E-8	.29156E-4	.81550E-3
2(5)	5.6	.11950E-3	.10233E-7	.95196E-4	.10920E-2
4(1)	35.0	.11574E-3	.95250E-3	.44520E-3	.17848E-3
4(2)	8.4	.11680E-3	.69912E-4	.16140E-2	.36666E-3
4(3)	1.5	.11133E-3	.11610E-5	.74780E-5	.20737E-3
9(1)	9.3	.11610E-3	.77947E-3	.30958E-2	.16013E-3
9(2)	20.4	.10429E-3	.17516E-4	.42568E-3	.33274E-3
9(3)	14.6	.11742E-3	.23481E-5	.20245E-3	.52333E-3
11(1)	27.4	.11750E-3	.14906E-2	.51031E-2	.14460E-3
11(2)	20.4	.10513E-3	.14101E-4	.28909E-3	.31755E-3
11(3)	9.0	.13211E-3	.34119E-5	.14798E-3	.49804E-3
13(1)	6.0	.17827E-3	.23326E-3	.11461E-2	.28379E-3
13(2)	5.0	.19295E-3	.44637E-4	.10750E-2	.61386E-3
13(3)	5.5	.15412E-3	.21375E-6	.13672E-3	.99577E-3

Table 15.

Parameters for Handsheet Specimens Made From Currency Pulp Less Fines (28-159-5).

Specimen No. and (Curve)	k	x_c	C_1	C_2	x_o
	N/m	m	N	N	m
1(1)	14.2	.15243E-3	.84484E-4	.58113E-3	.29395E-3
1(2)	13.1	.15060E-3	.48763E-5	.22533E-3	.57726E-3
1(3)	11.8	.15495E-3	.26148E-6	.13460E-3	.96744E-3
8(1)	12.4	.20390E-3	.12151E-2	.24635E-2	.14411E-3
8(2)	13.2	.19176E-3	.10270E-3	.52610E-3	.31327E-3
8(3)	11.6	.18555E-3	.66111E-5	.21302E-3	.64436E-3
10(1)	43.0	.11977E-3	.80178E-4	.54252E-3	.22900E-3
10(2)	9.9	.20492E-3	.16608E-3	.13967E-2	.43591E-3
10(3)	0.0	.22563E-3	.43567E-4	.11195E-2	.73246E-3
11(1)	69.4	.10362E-3	.10610E-2	.21655E-2	.73922E-4
11(2)	4.4	.17978E-3	.65611E-3	.39485E-2	.32267E-3
11(3)	0.4	.17871E-3	.45919E-4	.13073E-2	.59849E-3
13(1)	0.0	.25165E-3	.66651E-2	.10956E-1	.12507E-3
13(2)	9.6	.16088E-3	.17686E-3	.86927E-3	.25617E-3
13(3)	10.5	.14095E-3	.31879E-5	.20464E-3	.58662E-3
13(4)	5.2	.19190E-3	.48777E-5	.28541E-3	.78089E-3

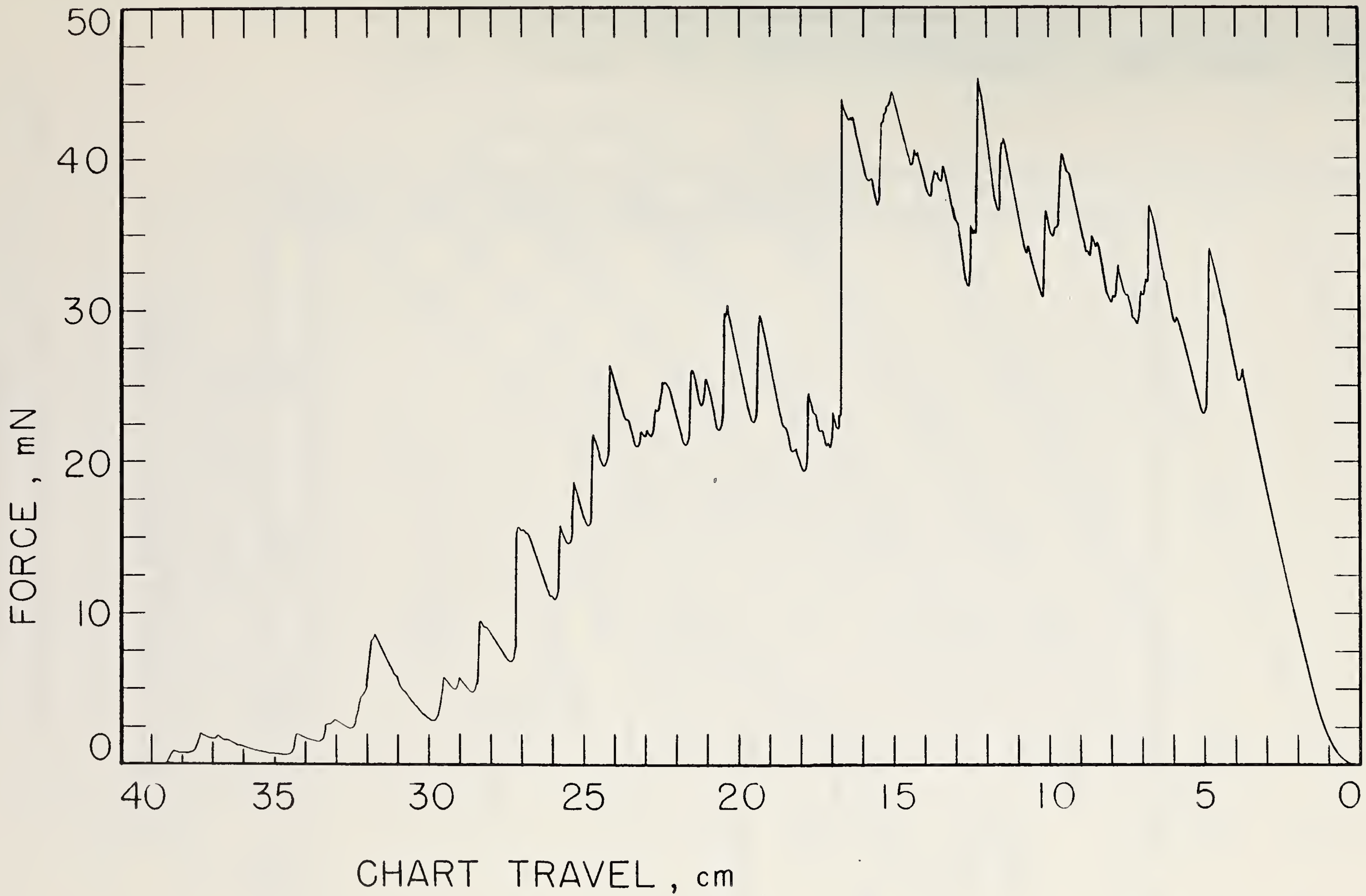

Figure 1. Force-elongation curve for a handsheet specimen of beaten Southern kraft pulp, 2.5 g/m^2 mass per unit area.

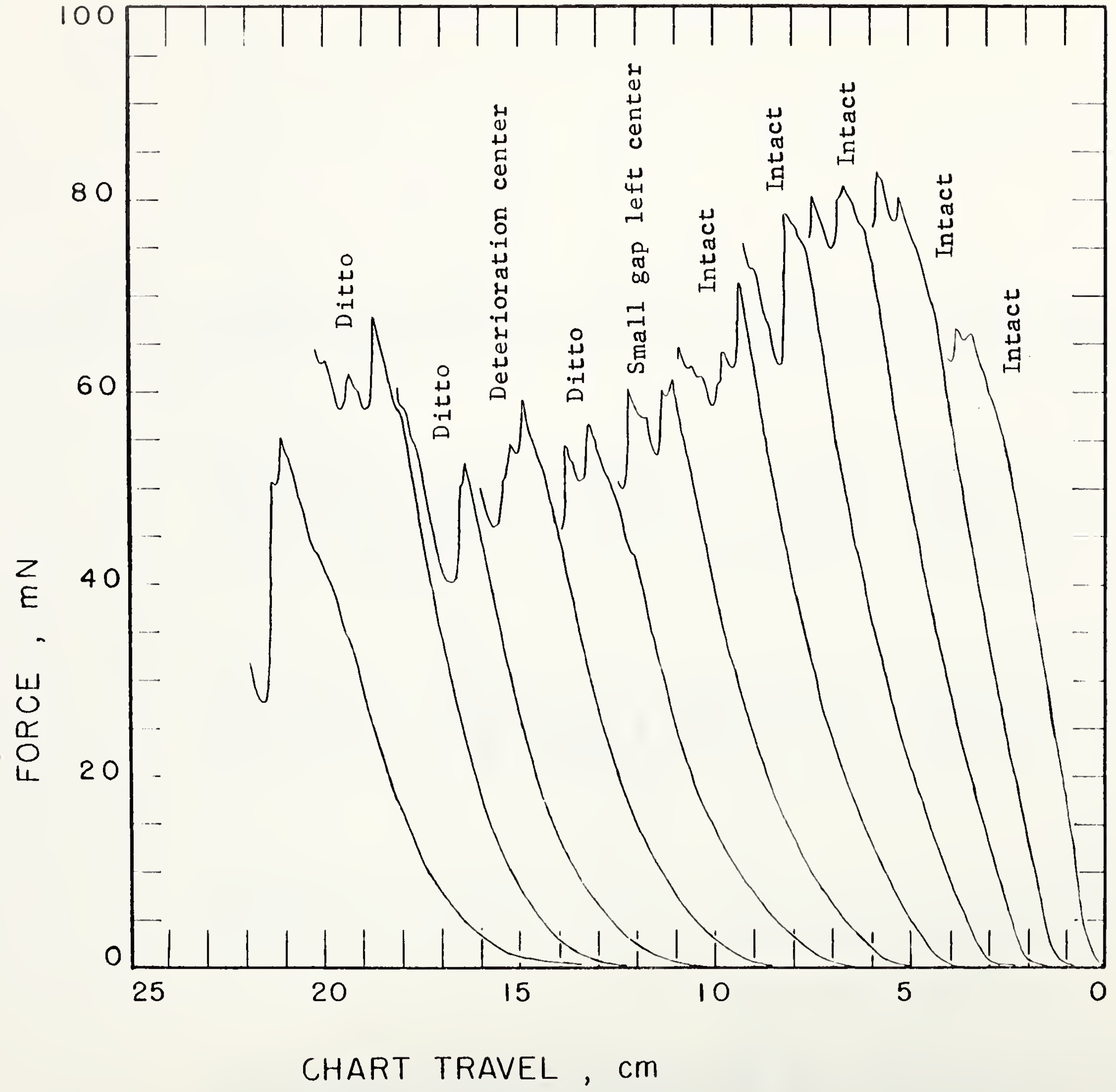

Figure 2. Force-elongation curve for a handsheet specimen of beaten Northern kraft pulp, 2.5 g/m^2 mass per unit area.

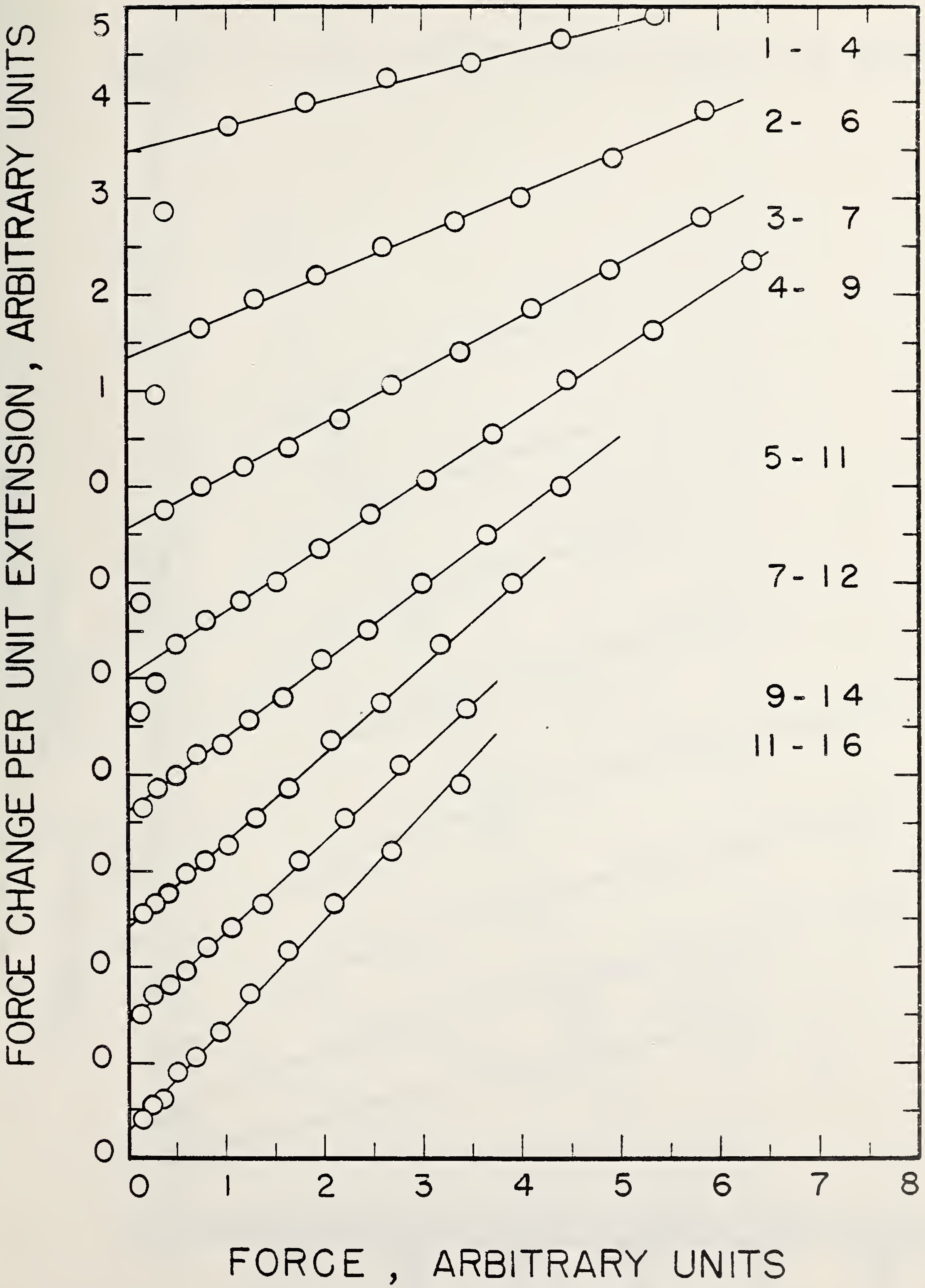

Figure 3. Plots of force change per unit extension versus force for reextension curves, figure 2.

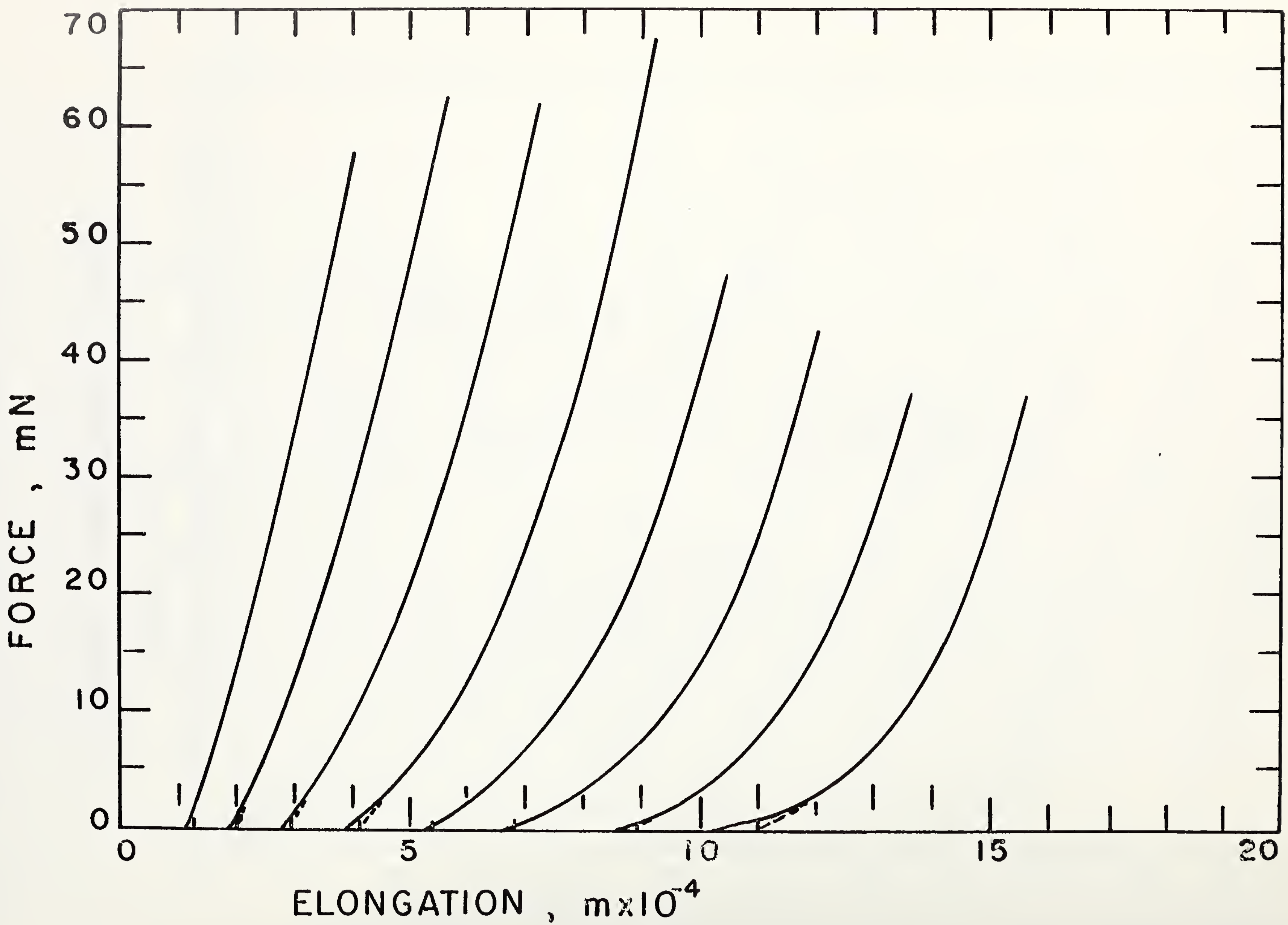

Figure 4. Comparison of test data with fitting curves:
solid lines, curves from test, figure 2;
dash lines, fitting curves from eq (4).

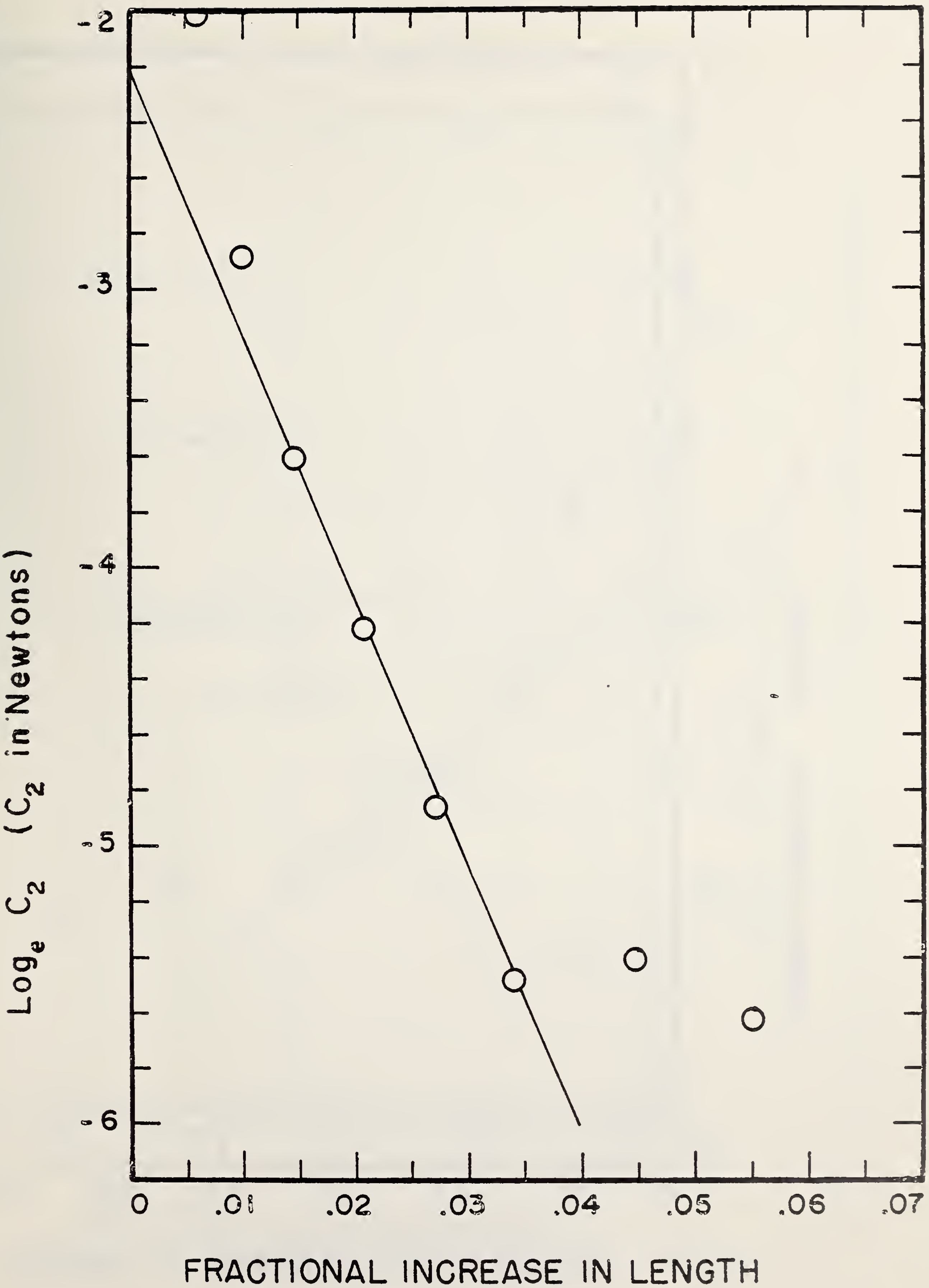

Figure 5. Plot of ln C_2 vs fractional increase in specimen length, data of table 3.

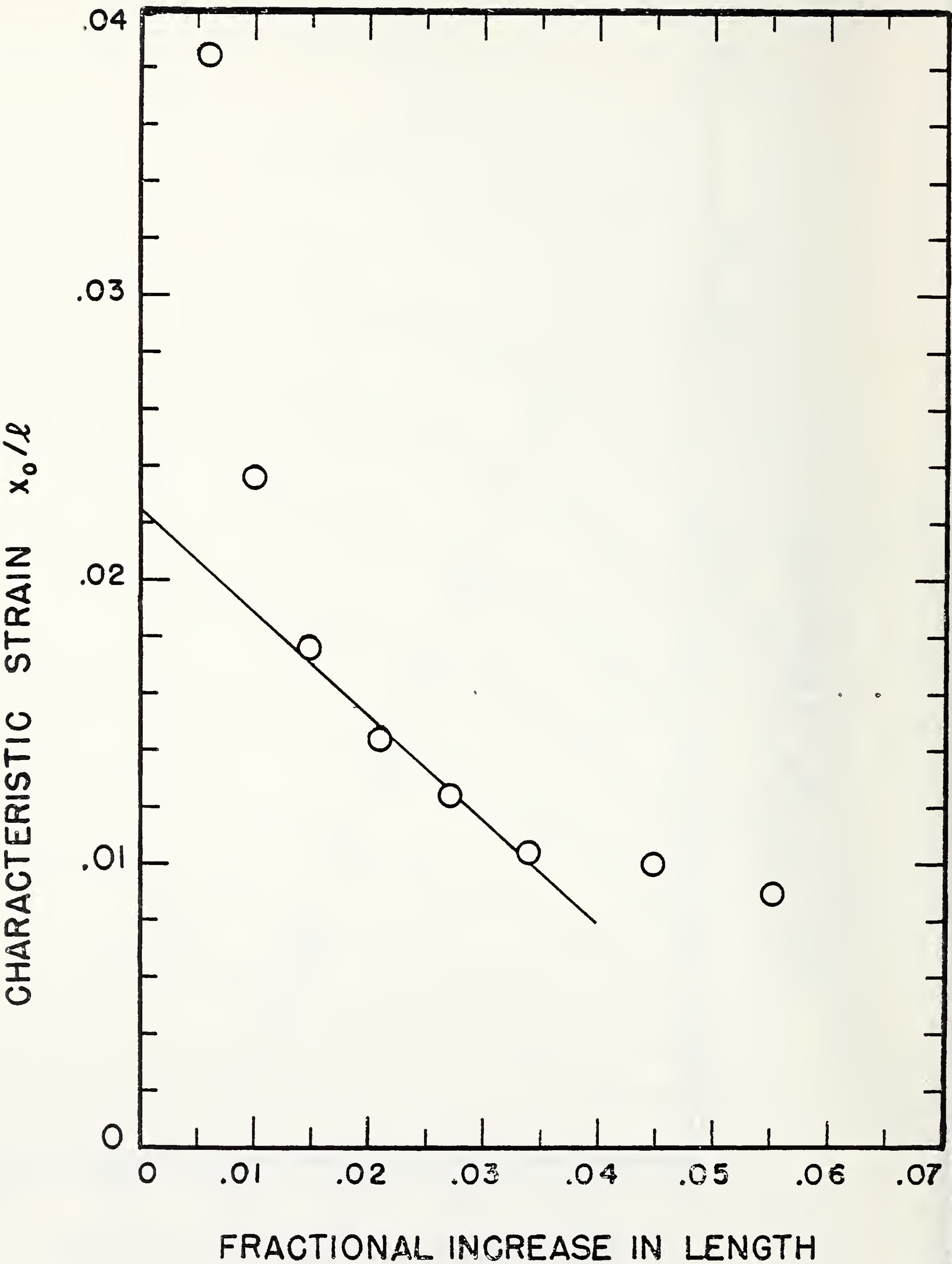

Figure 6. Plot of x_c/ℓ vs fractional increase in specimen length, data of table 3.

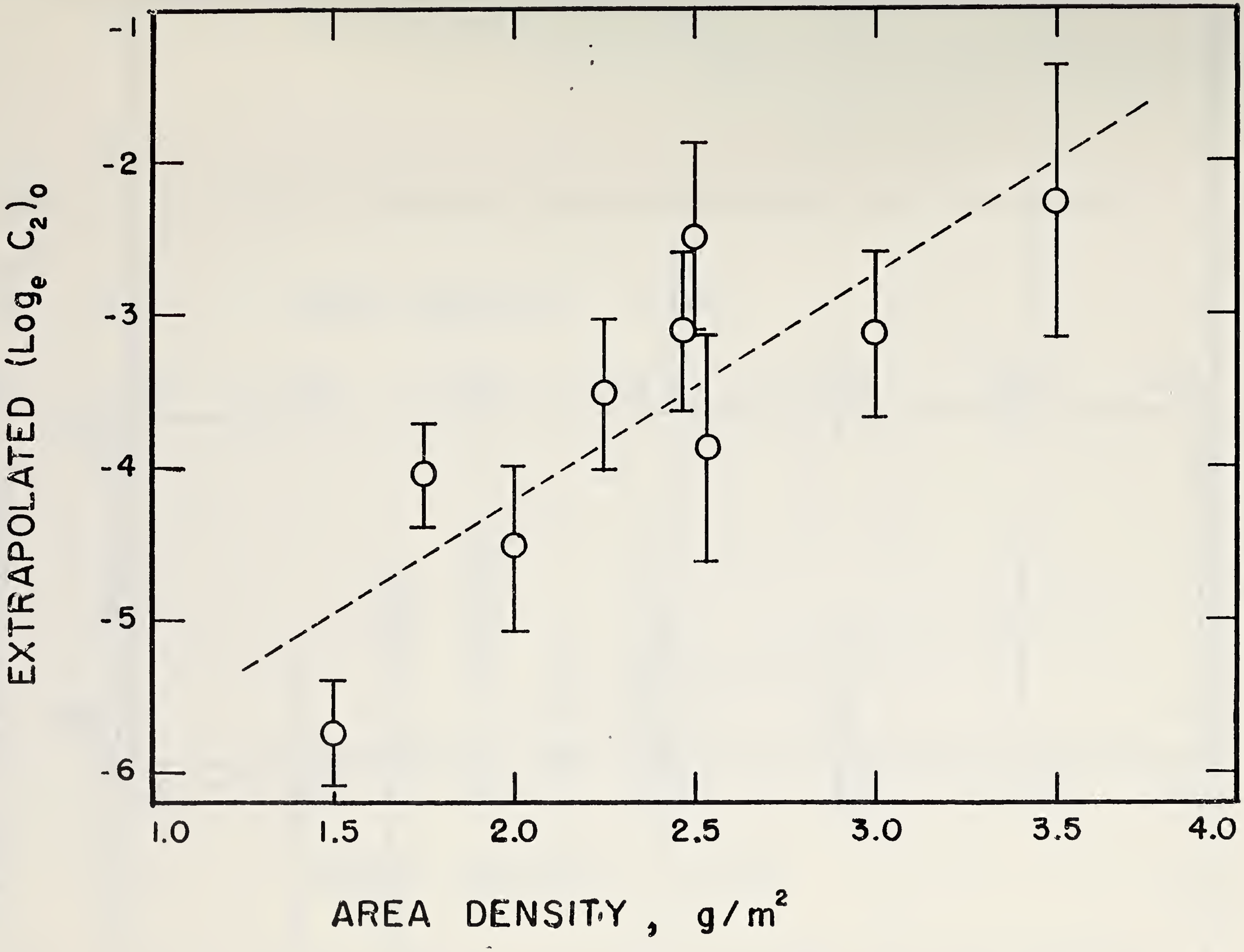

Figure 7. Plot of $(\ln C_2)_o$ as a function of area density for Northern Kraft pulp handsheets. C_2 is expressed in Newtons and density in g/m^2.

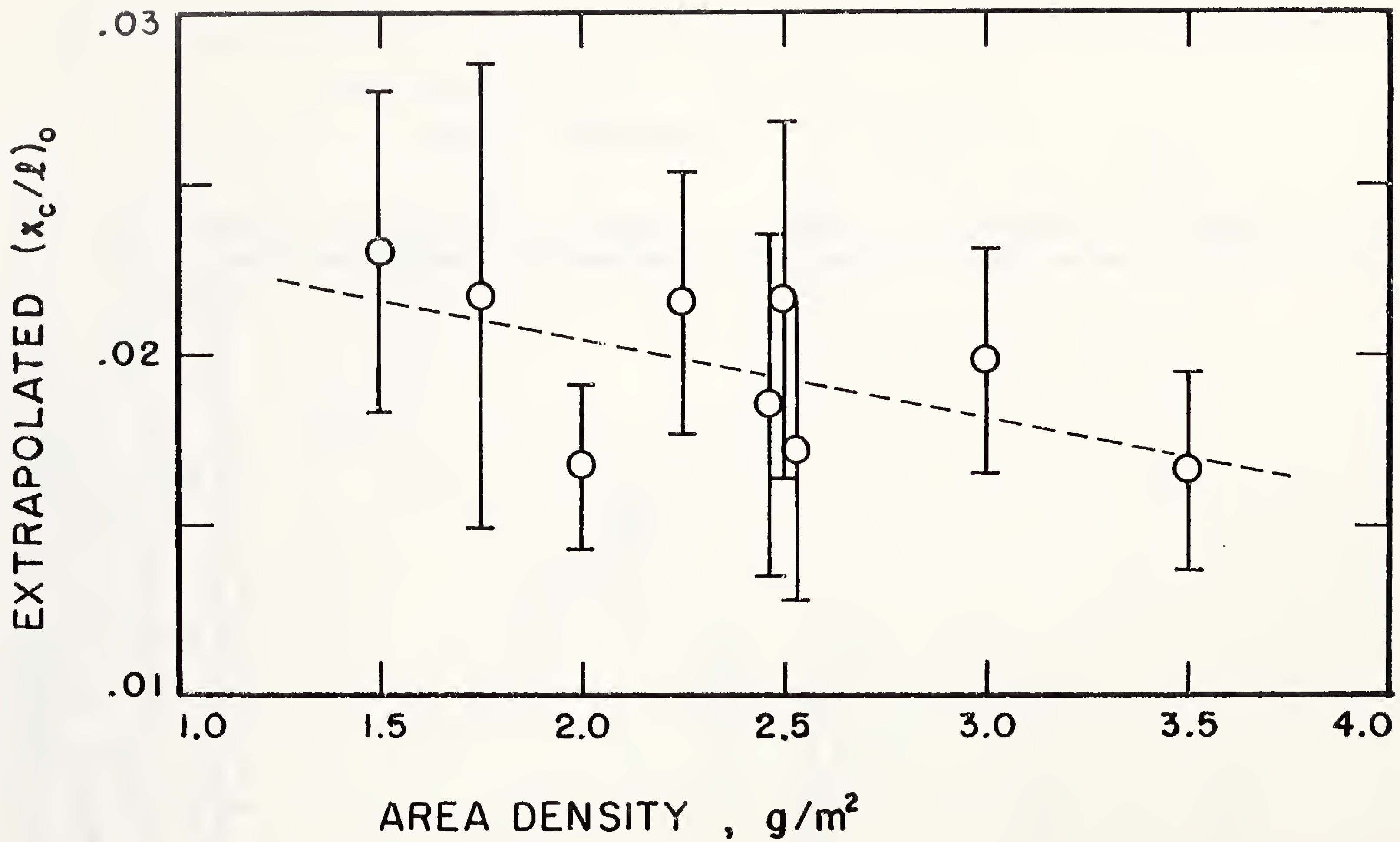

Figure 8. Plot of $(x_c/\ell)_o$ as a function of area density for Northern Kraft pulp handsheets.

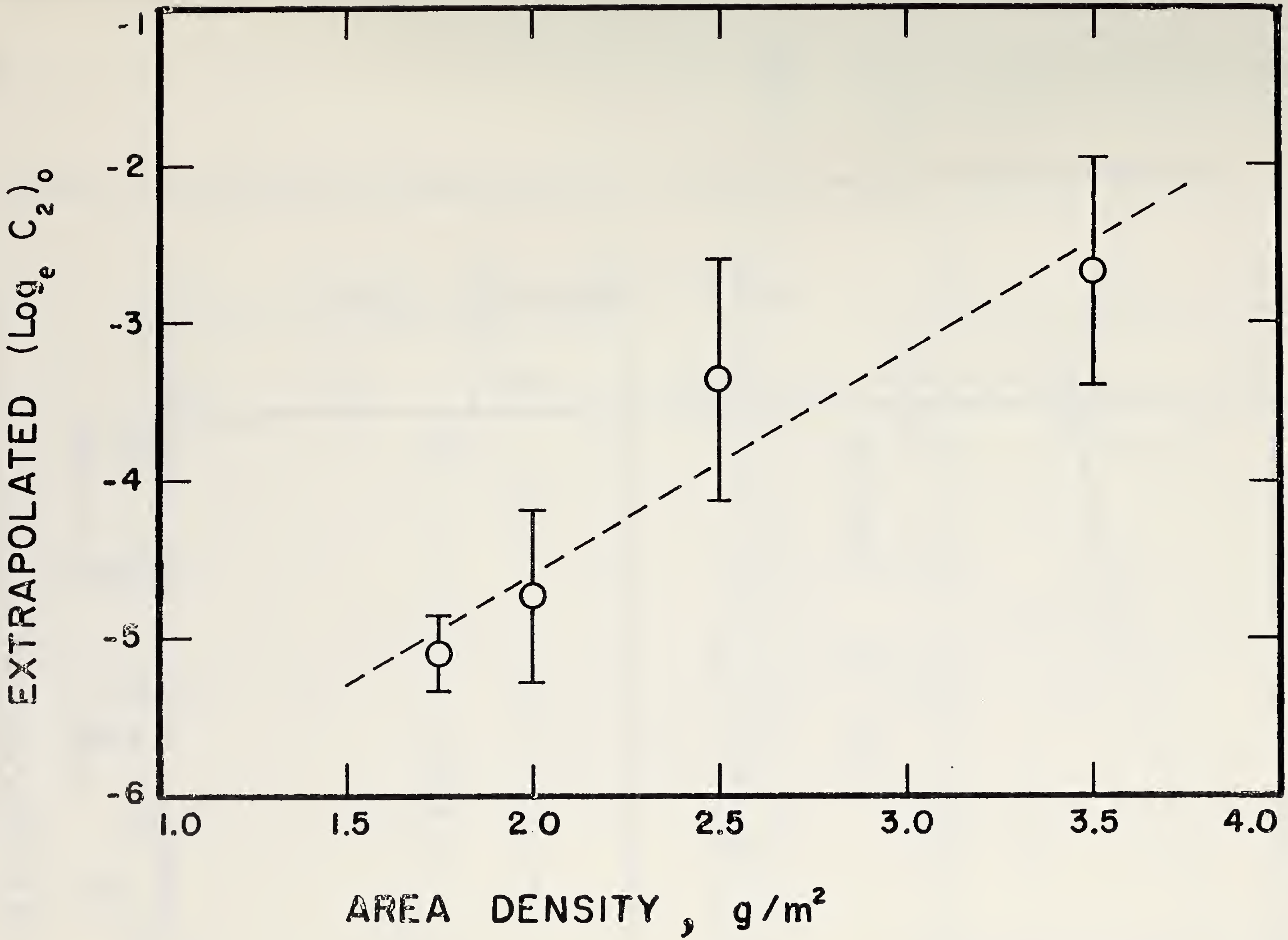

Figure 9. Plot of $(\ln C_2)_o$ as a function of area density for Southern Kraft pulp handsheets. C_2 is expressed in Newtons and density in g/m^2.

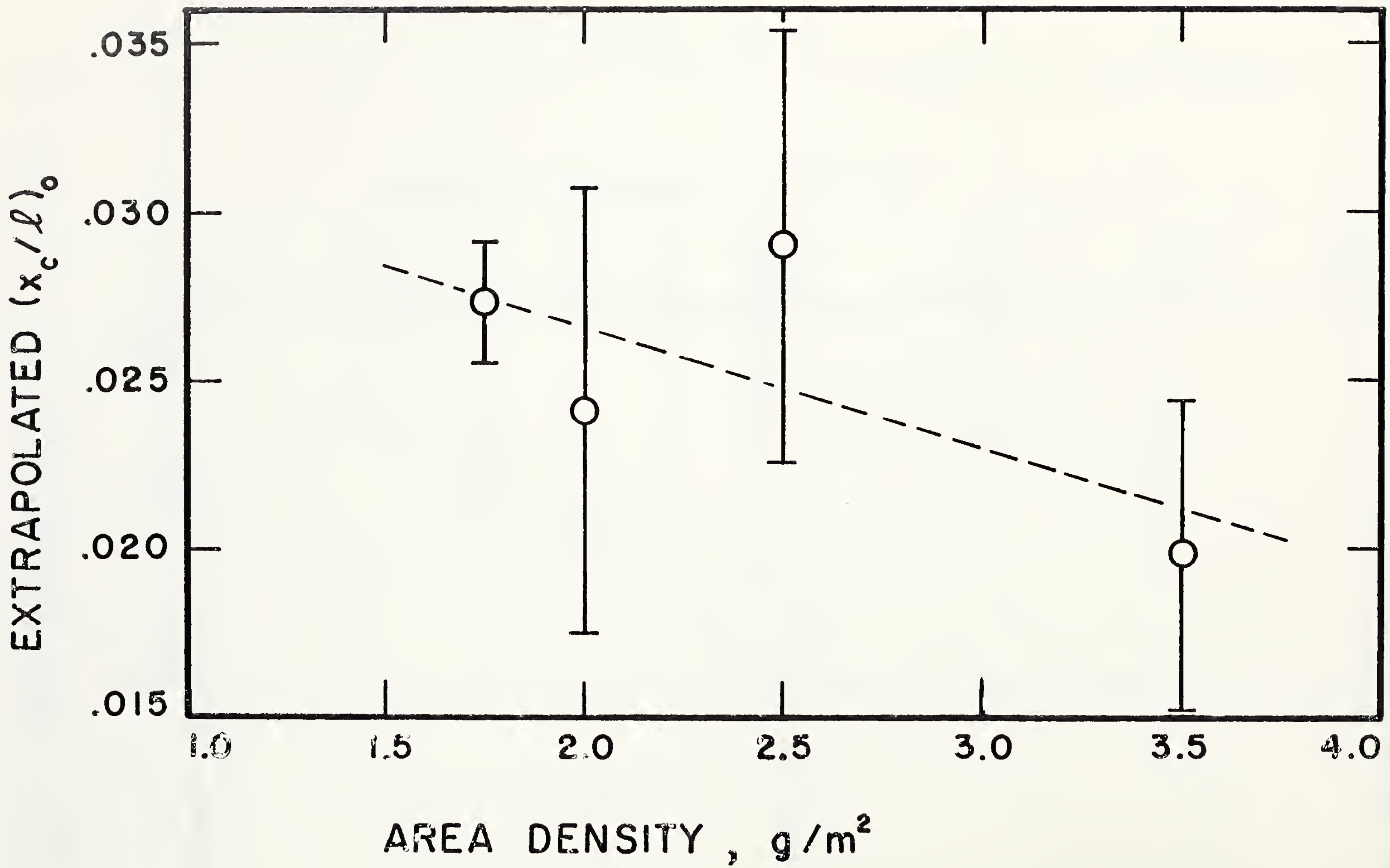

Figure 10. Plot of $(x_c/\ell)_o$ as a function of area density for Southern Kraft pulp handsheets.

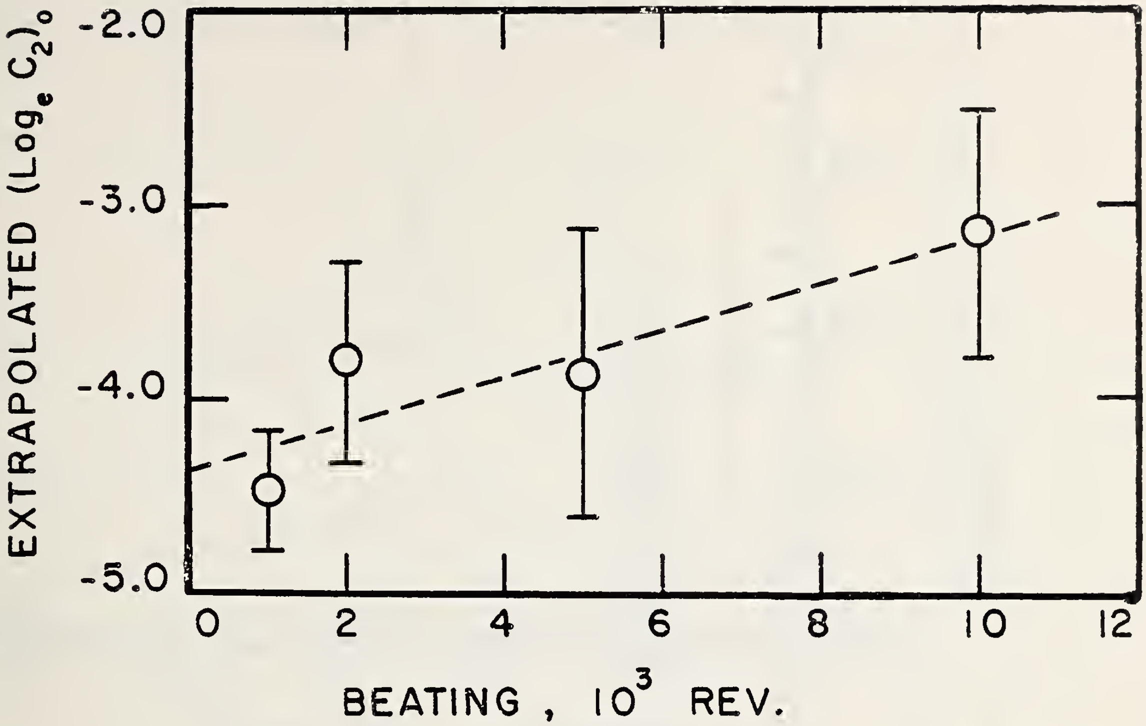

Figure 11. $(\ln C_2)_o$ as a function of beating for Northern Kraft pulp handsheets. C_2 is expressed in Newtons. Density of handsheets is 2.5 g/m^2.

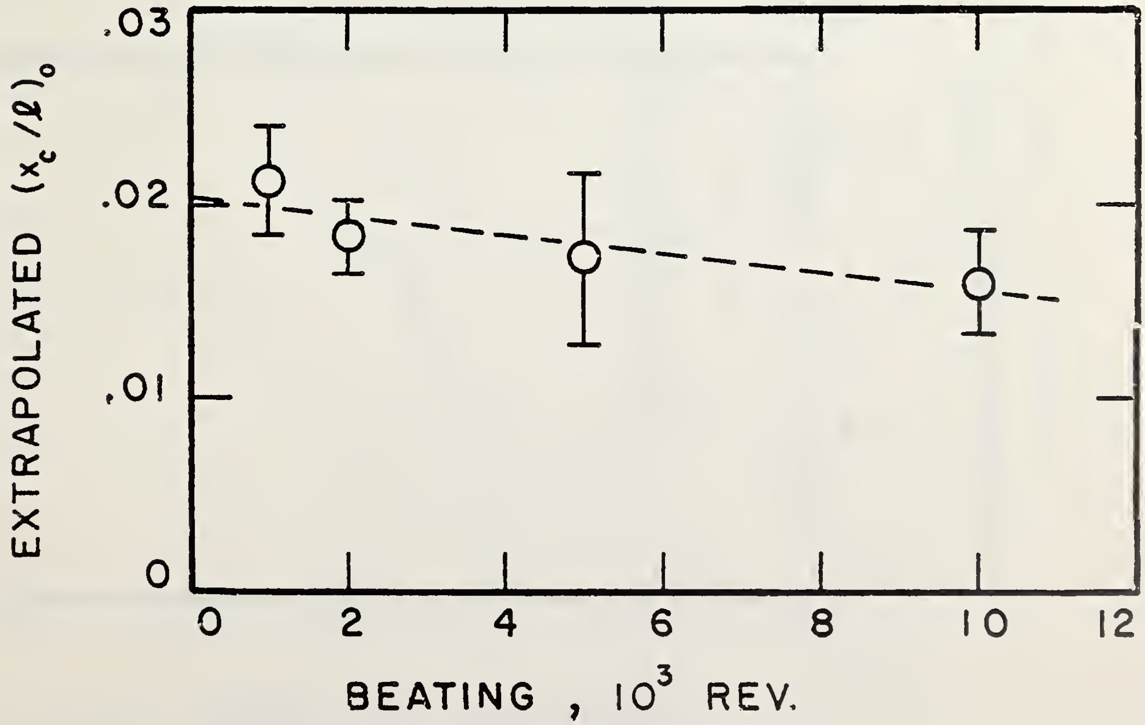

Figure 12. $(x_c/\ell)_o$ as a function of beating for Northern Kraft pulp handsheets.

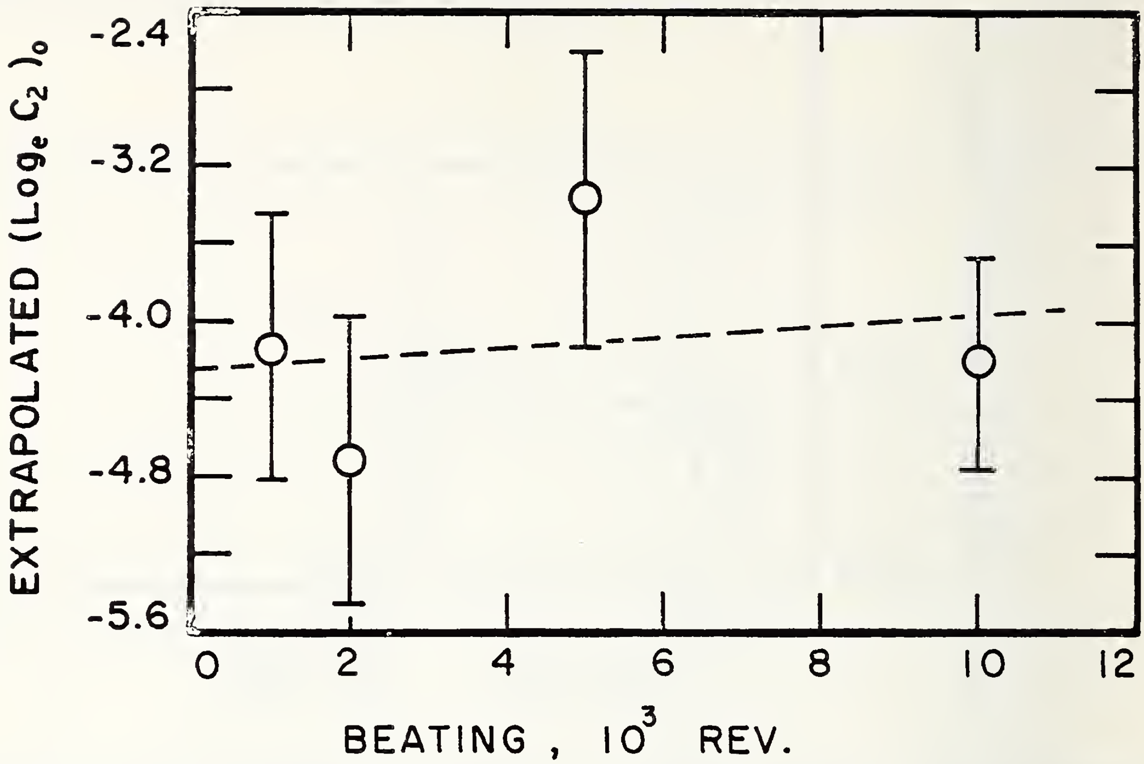

Figure 13. $(\ln C_2)_o$ as a function of beating for Southern Kraft pulp handsheets. C_2 is expressed in Newtons. Density of handsheets is 2.5 g/m^2.

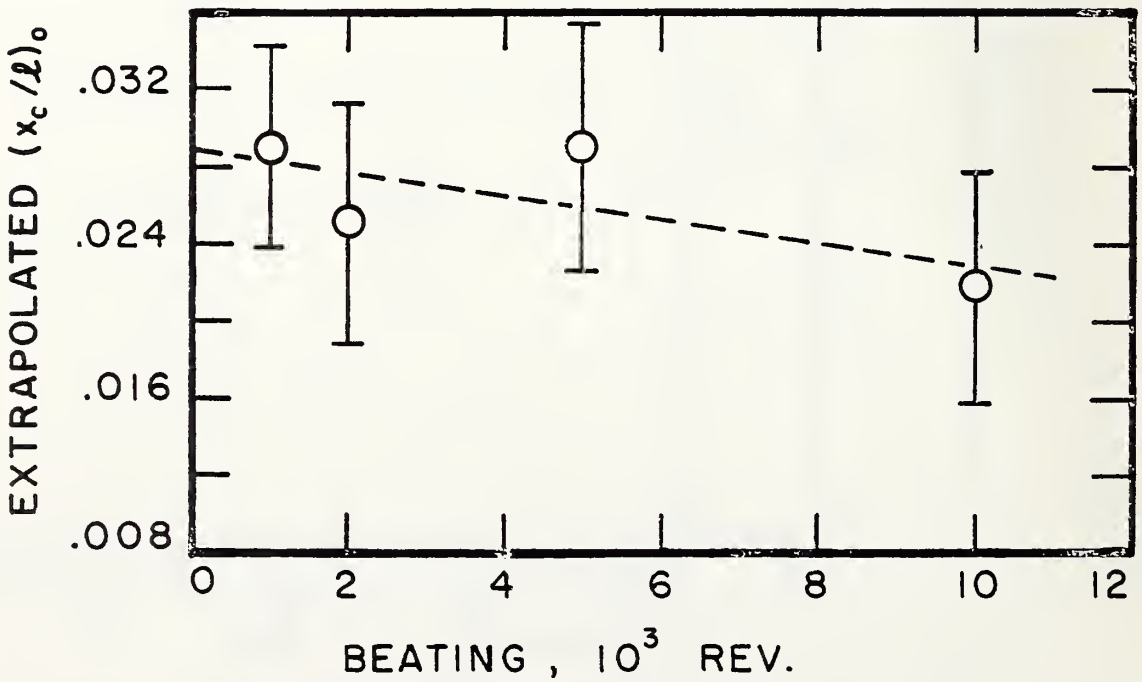

Figure 14. $(x_c/\ell)_o$ as a function of beating for Southern Kraft pulp handsheets.

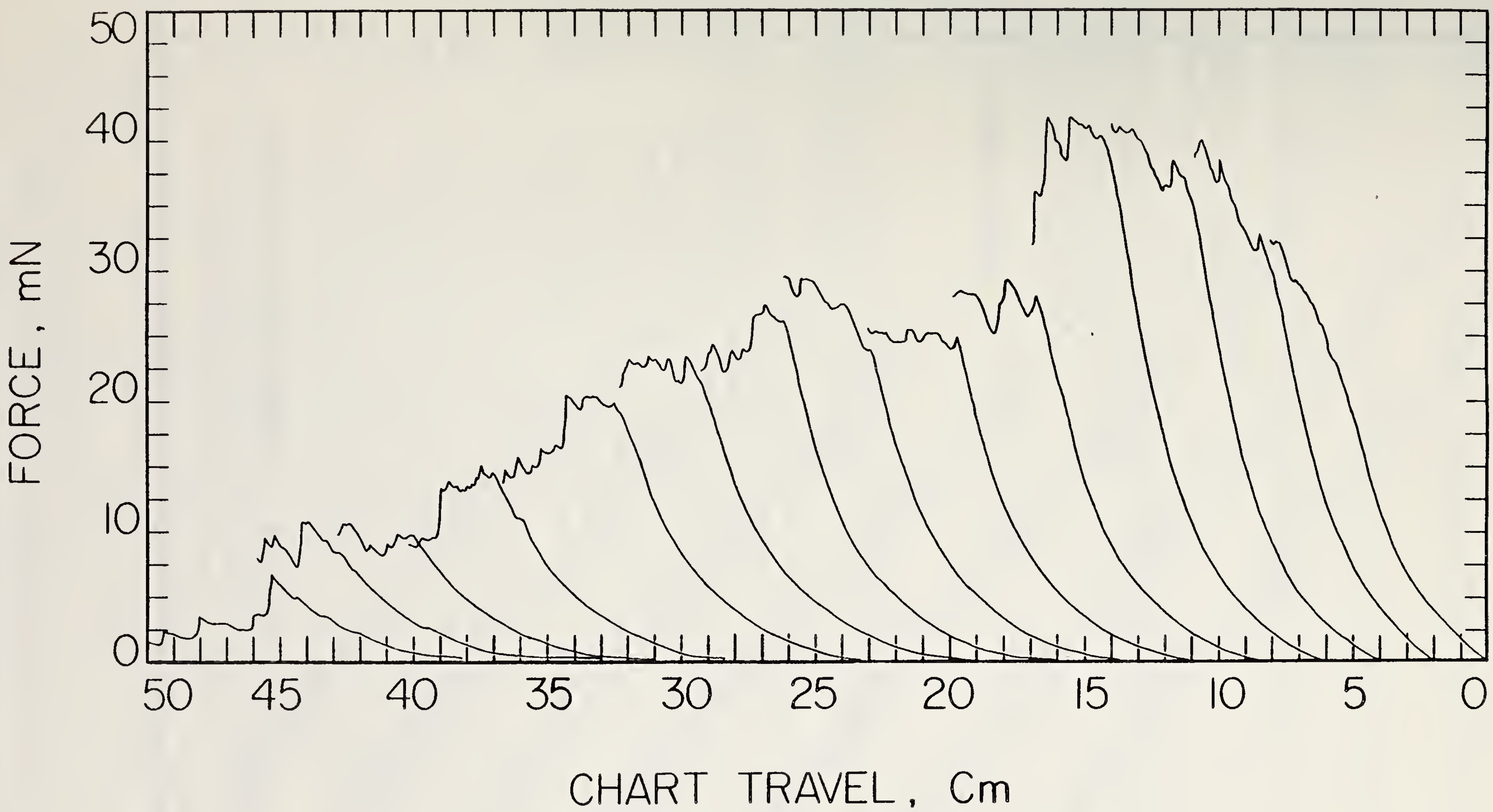

Figure 15. Force-elongation curves for a handsheet specimen of unfractionated currency pulp, 2.5 g/m^2 mass per unit area.

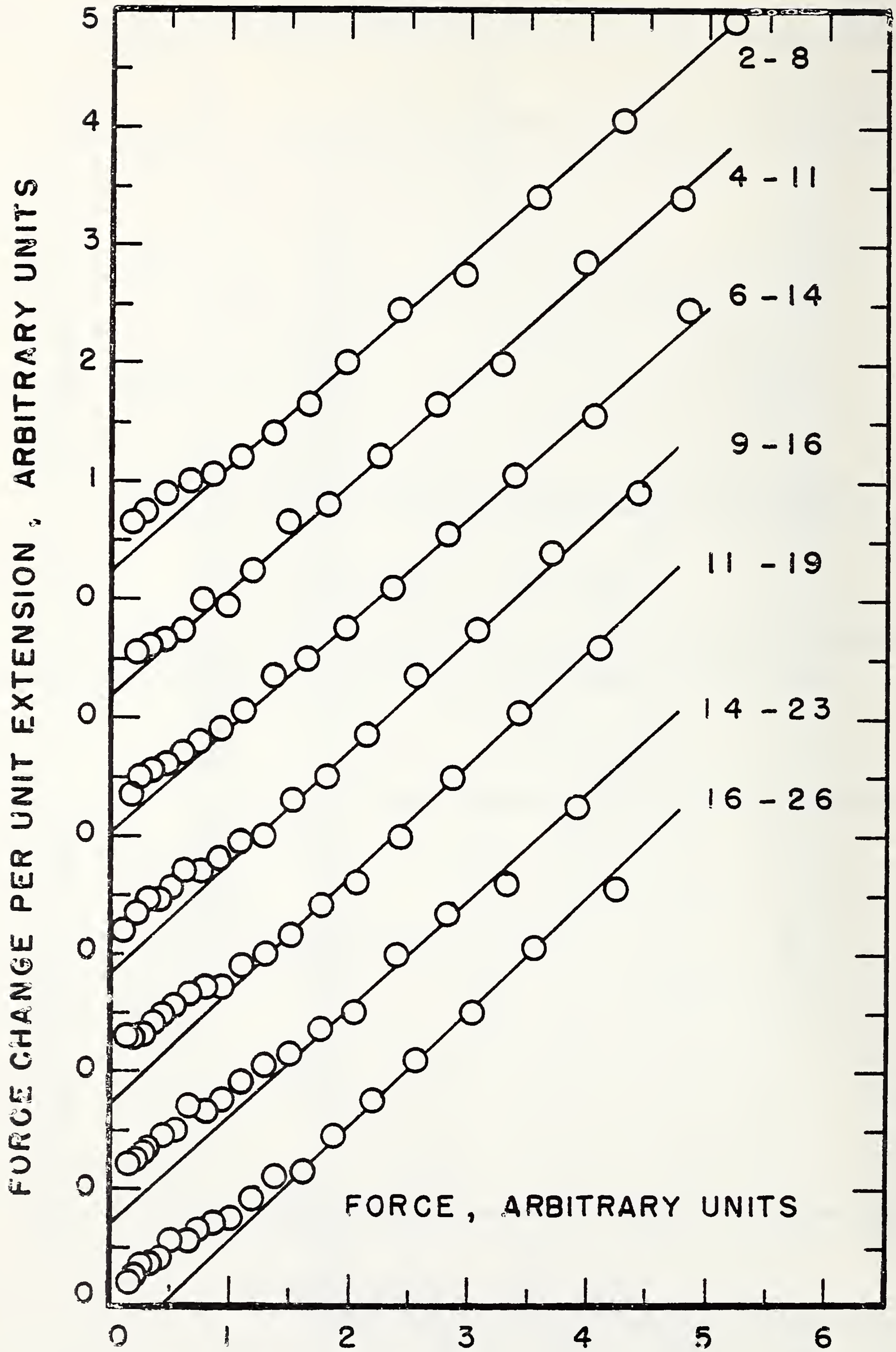

Figure 16. Plots of force change per unit extension versus force for reextension curves, figure 15.

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

Specimens of low-density open-web handsheets made from woodpulp and from a currency paper pulp were tested in a tensile tester to obtain data for characterizing interfiber bonding. During the tests the direction of extension was frequently reversed and the specimen reextended to obtain a series of force-elongation curves. For woodpulp specimens the force-elongation behavior was well represented by an exponential equation involving three parameters; a strength parameter, a parameter proportional to average mesh size of the fiber network and a parameter characterizing the unstrained length of the specimen. This force-elongation behavior is consistent with the concept of a network held together by simple bonds at fiber crossovers.

For currency pulp specimens the equation giving force as a function of elongation comprised the sum of an exponential term and a linear term. This behavior is consistent with the presence of an additional binding force, such as that provided by massive glued junctions or numerous bonds between microfibrils. Therefore the interfiber bonding cannot be well characterized in terms of an average bond strength. A force level parameter and a related elongation parameter applicable both to woodpulps and currency pulps are suggested as an alternate means of characterizing the strength of a test specimen.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Adhesion of paper fibers; bonding of paper fibers; paper fibers, bonding; paper pulps, characterization; paper, tensile tests.

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